CB – 167

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II Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2014 – 15 and Onwards) (Repeaters)
MATHEMATICS – II

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

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PART – A

1. Answer **any five** questions. **(5×2=10)**

   a) Define subgroup of a group.

   b) Prove that in a group $(G, *)$, $(a^{-1})^{-1} = a \ \forall a \in G$.

   c) Find the angle between the radius vector and the tangent to the curve $r = ae^{\theta\cot\alpha}$.

   d) Find the length of the polar subnormal at the point $\theta = \pi/6$ for the curve $r = a\cos 2\theta$.

   e) Find the asymptotes parallel to the co-ordinate axes for the curve $(x^2 + a^2)y = bx^2$.

   f) Find the length of an arc of the cycloid

   $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

   g) Show that the equation $(x^2 - 2xy + 3y^2)dx + (y^2 + 6xy - x^2)\,dy = 0$ is exact.

   h) Solve $P^2 - 5P + 6 = 0$. where $P = \dfrac{dy}{dx}$.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) If $(G, *)$ is a group, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}, \ \forall a, b \in G$.

   b) Prove that $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12.

   c) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ be two permutations of order 3 then find $f \circ g$ and $f^{-1} \circ g^{-1}$.

OR

3. a) Let G be the set of all rationals and $*$ be the binary operation on G defined by $a * b = \frac{ab}{7}$ $\forall$ a, b $\in$ G. Then prove that (G, $*$) is an abelian group.

   b) Show that cube roots of unity forms an abelian group under usual multiplication.

   c) A non-empty subset H of a group G is a subgroup of G if and only if,

      i) For every a, b $\in$ H $\Rightarrow$ $a * b \in H$.

      ii) For every a $\in$ H $\Rightarrow$ $a^{-1} \in H$.

PART – C

Answer **any two full** questions. **(2×15=30)**

4. a) With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$ for the curve $r = f(\theta)$.

   b) Show that the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$ intersect orthogonally.

   c) Find the circle of curvature of the curve $xy = a^2$ at (a, a).

OR

5. a) Find the envelope of the family of straight lines $y = ax + \frac{a}{\alpha}$, where α is a parameter.

   b) Derive the formula for radius of curvature in Cartesian form.

   c) Find the pedal equation of the curve $r^n = a^n \cos n\theta$.

6. a) Find all asymptotes of the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

   b) Find the surface area generated by revolving about y – axis of the curve $x = y^3$ from y = 0 to y = 2.

   c) Find the position and nature of the double points of the curve

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0.$$

OR

7. a) Find the length of the arc of the curve $y = \log(\sec x)$ from $x = 0$ to $x = \pi/3$.

b) Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.

c) Find the envelope of the family of lines $x \cos^3\alpha + y \sin^3\alpha = a$, where α is a parameter.

PART – D

Answer **one full** question. **(1×15=15)**

8. a) Solve : $\frac{dy}{dx} - 2y \tan x = y^2 \tan^2 x$.

b) Solve : $y = 2px + y^2p^3$.

c) Obtain the orthogonal trajectories of the family of parabolas $y = ax^2$, where 'a' is a parameter.

OR

9. a) Verify the condition of exactness and solve :

$$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5x^4)dy = 0.$$

b) Solve for y : $y = 2px - p^2$.

c) Find the general and singular solution of

$$p^2(x^2 - a^2) - 2pxy + y^2 + a^2 = 0.$$