IV Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2021-22 and Onwards) (Repeaters)
MATHEMATICS – IV

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** Parts.

PART – A

1. Answer **any five** questions :   (5×2=10)

   a) Define a normal subgroup.

   b) Define homomorphism of groups.

   c) Obtain half range sine series $f(x) = 2x - 1$ in the interval $(0, 1)$.

   d) Define Fourier series.

   e) Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$.

   f) Evaluate $\lim\limits_{x \to 0} x^x$.

   g) Solve $y'' - 6y' + 13y = 0$.

   h) Find the particular integral of $y'' - 6y' + 9y = 3e^{-4x}$.

PART – B

2. Answer **any two** questions :   (2×5=10)

   a) Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H \; \forall g \in G, h \in H$.

   b) Prove that the product of any two normal subgroups of a group is again a normal subgroup of the same.

   c) State and prove fundamental theorem of Homomorphism.

PART – C

3. Answer **any two** questions :   (2×5=10)

   a) Obtain the Fourier expansion of the function $f(x) = e^x$ in $(-\pi, \pi)$.

   b) Obtain the Fourier half range cosine series for the function $f(x) = \sin x$ in $(0, \pi)$.

   c) Obtain the Fourier series of $f(x) = \dfrac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dots$

PART – D

4. Answer **any three** questions : (3×5=15)

   a) Discuss the continuity of the functions $f(x) = \begin{cases} 1+x & \text{for} \quad x \le 2 \\ 5-x & \text{for} \quad x > 2 \end{cases}$ at x = 2.

   b) State and prove Rolle's theorem.

   c) Expand tanx upto the term containing x^5 using Maclaurin's expansion.

   d) Find the extreme values of $f(x, y) = 2x^2 - xy + y^2 + 7x$.

   e) Obtain the Taylor's expansion of $f(x, y) = x^2y + 3y - 4$ in power of (x – 1) and (y + 2).

PART – E

5. Answer **any three** questions : (3×5=15)

   a) Solve : $(D^2 - 5D + 6)y = e^{4x}$.

   b) Solve : $x^2y'' - xy' + 2y = x \log x$.

   c) Solve : $y'' + 2y' + 4y = e^x \sin x$.

   d) Solve : $xy'' + (2x^2 - 1)y' - 24x^3y = 4x^3 \sin x^2$ (x > 0) by using the transformation $z = x^2$.

   e) Solve by the method of variation of parameter $y'' + a^2y = \sec ax$.

PART – F

6. Answer **any two** questions : (2×5=10)

   a) Sketch the graph of triangular wave and explain it.

   b) Obtain the Fourier series expansion of the following anti symmetrical square wave.
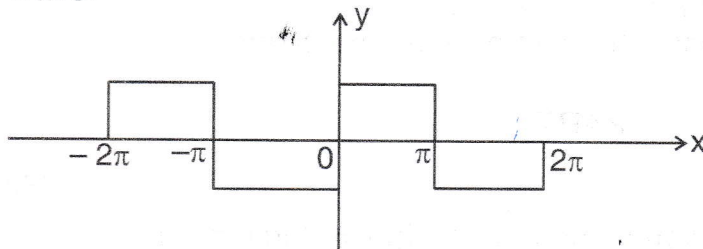
   c) Sketch the graph of sawtooth wave and explain it.