NP – 190

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II Semester B.Sc. Examination, August/September 2023
(NEP Scheme)
MATHEMATICS
DSC – 2.1 : Algebra – II and Calculus – II

Time : 2½ Hours

Max. Marks : 60

PART – A

I. Answer **any four** of the following : **(4×2=8)**

1) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$ find $f^{-1}og$.

2) Show that $f : (Z, +) \to (2Z, +)$ defined by $f(x) = 2x$ is a homomorphism.

3) Find the order of each element of the multiplicative group $G = \{1, -1, i, -i\}$.

4) If $u = x^2 - 2y$, $v = x + y$ show that $\frac{\partial(u, v)}{\partial(x, y)} = 2(x + 1)$.

5) Write the necessary condition for $f(x, y)$ to have an extremum at (a, b).

6) Evaluate $\int_0^a \int_0^a \int_0^a e^{x+y+z}\, dx\, dy\, dz$.

PART – B

II. Answer **any four** of the following : **(4×5=20)**

7) In a group G, if $O(a) = n \ \forall\ a \in G$, $d = (n, m)$, then prove that $O(a^m) = \frac{n}{d}$.

8) Find all the right and left cosets of the subgroup $H = \{0, 2, 4\}$ in $(Z_6, +_6)$.

9) If 'a' is a generator of a cyclic group G, then prove that a^{-1} is also a generator.

10) With usual notation prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

11) By changing the order of integration evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$.

12) Evaluate $\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2)\, dx\, dy\, dz$.

P.T.O.

PART – C

III. Answer **any four** of the following : **(4×8=32)**

13) Define a normal subgroup of a group G and prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H$, $\forall\ g \in G$.

14) State and prove the fundamental theorem of homomorphism on groups.

15) Define coset of a subgroup H in group G and prove that any two right (left) cosets of a subgroup H of a group G are either disjoint or identical.

16) Expand $f(x, y) = e^x \sin y$ by Taylor's theorem in powers of x and y as far as terms of fifth degree.

17) If $x = u(1 - v)$, $y = uv$, calculate $J = \frac{\partial(x, y)}{\partial(u, v)}$ and $J' = \frac{\partial(u, v)}{\partial(x, y)}$. Also verify $J.J' = 1$.

18) Evaluate $\iiint_R xyz\ dx\ dy\ dz$, where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.