



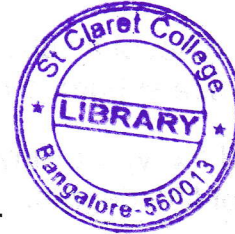
**II Semester B.Sc. Examination, August/September 2023  
(NEP Scheme)  
MATHEMATICS**

**DSC – 2.1 : Algebra – II and Calculus – II**

Time : 2½ Hours

Max. Marks : 60

## PART – A

I. Answer **any four** of the following :

(4×2=8)

1) If  $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$  and  $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$  find  $f^{-1}og$ .

2) Show that  $f : (Z, +) \rightarrow (2Z, +)$  defined by  $f(x) = 2x$  is a homomorphism.

3) Find the order of each element of the multiplicative group  $G = \{1, -1, i, -i\}$ .

4) If  $u = x^2 - 2y$ ,  $v = x + y$  show that  $\frac{\partial(u, v)}{\partial(x, y)} = 2(x + 1)$ .

5) Write the necessary condition for  $f(x, y)$  to have an extremum at  $(a, b)$ .

6) Evaluate  $\int_0^a \int_0^a \int_0^a e^{x+y+z} dx dy dz$ .

## PART – B

II. Answer **any four** of the following :

(4×5=20)

7) In a group  $G$ , if  $O(a) = n \forall a \in G$ ,  $d = (n, m)$ , then prove that  $O(a^m) = \frac{n}{d}$ .

8) Find all the right and left cosets of the subgroup  $H = \{0, 2, 4\}$  in  $(Z_6, +_6)$ .

9) If 'a' is a generator of a cyclic group  $G$ , then prove that  $a^{-1}$  is also a generator.

10) With usual notation prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$ .

11) By changing the order of integration evaluate  $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ .

12) Evaluate  $\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2) dx dy dz$ .

P.T.O.



## PART – C

III. Answer **any four** of the following :**(4×8=32)**

- 13) Define a normal subgroup of a group  $G$  and prove that a subgroup  $H$  of a group  $G$  is normal if and only if  $gHg^{-1} = H, \forall g \in G$ .
  - 14) State and prove the fundamental theorem of homomorphism on groups.
  - 15) Define coset of a subgroup  $H$  in group  $G$  and prove that any two right (left) cosets of a subgroup  $H$  of a group  $G$  are either disjoint or identical.
  - 16) Expand  $f(x, y) = e^x \sin y$  by Taylor's theorem in powers of  $x$  and  $y$  as far as terms of fifth degree.
  - 17) If  $x = u(1 - v), y = uv$ , calculate  $J = \frac{\partial(x, y)}{\partial(u, v)}$  and  $J' = \frac{\partial(u, v)}{\partial(x, y)}$ . Also verify  $J \cdot J' = 1$ .
  - 18) Evaluate  $\iiint_R xyz \, dx \, dy \, dz$ , where  $R$  is the positive octant of the sphere  $x^2 + y^2 + z^2 = a^2$ .
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