



CB – 166

21

II Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (Repeaters) (2020 – 21 and Onwards)
MATHEMATICS (Paper – II)

Time : 3 Hours

Max. Marks : 70

Instruction : Answer *all* the Parts.

PART – A

Answer **any five** questions.

(5×2=10)

1. a) On \mathbb{Q}^+ , $*$ is defined as $a * b = \frac{ab}{4}$, $\forall a, b \in \mathbb{Q}^+$ show that the binary operation $*$ is associative.
- b) Define subgroup of a group and give an example.
- c) Find the polar subnormal of the curve $r = a\theta$.
- d) For the curve $r = a(1 - \cos\theta)$, find the angle ϕ at $\theta = \frac{\pi}{3}$.
- e) Find $\frac{ds}{dx}$ for $ay^2 = x^3$.
- f) Find the integrating factor of $\frac{dy}{dx} + y = e^{-x}$.
- g) Find the length of the curve $4y^2 = x^2$ between $x = 0$ and $x = 5$.
- h) Solve $p^2 - 5p - 6 = 0$ where $p = \frac{dy}{dx}$.



PART – B

Answer **any two** questions.

(2×5=10)

2. Prove that fourth roots of unity forms an abelian group under multiplication.
3. If G be a set of rationals except -1 and $*$ is the binary operation on G defined by $a * b = a + b + ab$, prove that $(G, *)$ is a group.
4. Prove that $H = \{0, 2, 4\}$ is a subgroup of the group $G = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6.

P.T.O.



PART – C

Answer **any three** questions.**(3×5=15)**

5. Show that the curves $r = a \sec^2 \frac{\theta}{2}$, $r = b \operatorname{cosec}^2 \frac{\theta}{2}$ cut orthogonally.
6. Show that the pedal equation of cardioid $r = a(1 - \cos\theta)$ is $2ap^2 = r^3$.
7. Find the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.
8. Find all asymptotes of $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0$.
9. Find the envelope of the family of circles $(x - \alpha)^2 + y^2 = r^2$ where ' α ' is a parameter.

PART – D

Answer **any two** questions.**(2×5=10)**

10. Find the area of cardioid $r = a(1 + \cos\theta)$.
11. Find the surface area generated by revolving the curve $x = y^2$ about y-axis from $y = 0$ to $y = 2$.
12. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis.

PART – E

Answer **any three** questions.**(3×5=15)**

13. Solve $\frac{dy}{dx} + y \sec x = \tan x$.
14. Verify for exactness and hence solve $(2xy + 3y) dx + (x^2 + 3x) dy = 0$.
15. Solve $xp^2 + (y - x)p - y = 0$.
16. Solve $yp^2 - 2xp + y = 0$.
17. Find the orthogonal trajectory of family of curves $xy = c^2$.



PART – F

Answer **any two** questions.

(2×5=10)

18. Find the surface area of the reel, when the arc of the parabola $y^2 = 4x$ between the points (1, 2) and (4, 4) is revolved about the x-axis.
 19. A cake is removed from an oven at 180°F and placed in a room with 70°F . Three minutes later it cooled to 60°F . Find its temperature after 10 minutes.
 20. Find the equation of curve passing through the point (0, 2), given that sum of co-ordinates at any point on the curve exceeds the magnitude of slope of the tangent to the curve at the point by 5.
-