



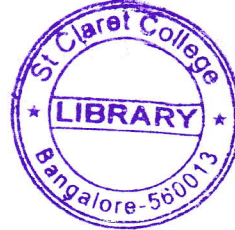
II Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2014 – 15 and Onwards) (Repeaters)
MATHEMATICS – II

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.

PART – A

1. Answer **any five** questions.

(5×2=10)

- Define subgroup of a group.
- Prove that in a group $(G, *)$, $(a^{-1})^{-1} = a \forall a \in G$.
- Find the angle between the radius vector and the tangent to the curve $r = ae^{\theta \cot \alpha}$.
- Find the length of the polar subnormal at the point $\theta = \frac{\pi}{6}$ for the curve $r = a \cos 2\theta$.
- Find the asymptotes parallel to the co-ordinate axes for the curve $(x^2 + a^2)y = bx^2$.
- Find the length of an arc of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$.
- Show that the equation $(x^2 - 2xy + 3y^2)dx + (y^2 + 6xy - x^2)dy = 0$ is exact.
- Solve $P^2 - 5P + 6 = 0$. where $P = \frac{dy}{dx}$.

PART – B

Answer **one full** question.

(1×15=15)

- If $(G, *)$ is a group, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, $\forall a, b \in G$.
 - Prove that $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12.
 - If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ be two permutations of order 3 then find $f \circ g$ and $f^{-1} \circ g^{-1}$.

OR

P.T.O.



3. a) Let G be the set of all rationals and $*$ be the binary operation on G defined by $a * b = \frac{ab}{7} \forall a, b \in G$. Then prove that $(G, *)$ is an abelian group.
- b) Show that cube roots of unity forms an abelian group under usual multiplication.
- c) A non-empty subset H of a group G is a subgroup of G if and only if,
- For every $a, b \in H \Rightarrow a * b \in H$.
 - For every $a \in H \Rightarrow a^{-1} \in H$.

PART – C

Answer **any two full** questions.

(2×15=30)

4. a) With usual notations prove that $\tan \phi = r \frac{d\theta}{dr}$ for the curve $r = f(\theta)$.
- b) Show that the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$ intersect orthogonally.
- c) Find the circle of curvature of the curve $xy = a^2$ at (a, a) .

OR

5. a) Find the envelope of the family of straight lines $y = ax + \frac{a}{\alpha}$, where α is a parameter.
- b) Derive the formula for radius of curvature in Cartesian form.
- c) Find the pedal equation of the curve $r^n = a^n \cos n\theta$.
6. a) Find all asymptotes of the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

- b) Find the surface area generated by revolving about y – axis of the curve $x = y^3$ from $y = 0$ to $y = 2$.

- c) Find the position and nature of the double points of the curve

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0.$$

OR



7. a) Find the length of the arc of the curve $y = \log(\sec x)$ from $x = 0$ to $x = \pi/3$.
- b) Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.
- c) Find the envelope of the family of lines $x \cos^3\alpha + y \sin^3\alpha = a$, where α is a parameter.

PART – D

Answer **one full** question.

(1×15=15)

8. a) Solve : $\frac{dy}{dx} - 2y \tan x = y^2 \tan^2 x$.
- b) Solve : $y = 2px + y^2 p^3$.
- c) Obtain the orthogonal trajectories of the family of parabolas $y = ax^2$, where 'a' is a parameter.

OR

9. a) Verify the condition of exactness and solve :
 $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5x^4)dy = 0$.
- b) Solve for y : $y = 2px - p^2$.
- c) Find the general and singular solution of
 $p^2(x^2 - a^2) - 2pxy + y^2 + a^2 = 0$.
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