NP – 193

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IV Semester B.Sc. Examination, August/September 2023
(NEP)
MATHEMATICS – IV
DSC 4.1 : Partial Differential Equations and Integral Transforms

Time : 2½ Hours | Max. Marks : 60

Instruction *: Answer* ***all*** *Parts.*

PART – A

I. Answer **any four** of the following : **(4×2=8)**

1) Solve $p^2 + q^2 = 3$.
2) Solve $p - x^2 = q + y^2$.
3) Solve $[D^2 - 2DD' + D'^2]\, z = 0$.
4) Find the Laplace transform of $e^{3t}\,[3\sin 4t - 9\cos 4t]$.
5) Find $L^{-1}\left\{\dfrac{(s^2-3)^2}{s^5}\right\}$.
6) Find a_0 in the Fourier series of $f(x) = x^2$ in $(0, 2)$.

PART – B

II. Answer **any four** of the following : **(4×5=20)**

7) Form the PDE by eliminating the arbitrary function from $z = f(x + ay) + g(x - ay)$.
8) Solve $(mz - ny)\,p + (nx - lz)\,q = ly - mx$.
9) Solve $[2D' - DD' - 3D'^2]z = 5e^{x-y}$.
10) Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$.
11) Find $L^{-1}\left\{\dfrac{3s+2}{s^2+6s+9}\right\}$.
12) Find the half range Fourier cosine series for the function $f(x) = (x-1)^2$ in $(0, 1)$.

P.T.O.

PART – C

III. Answer **any four** of the following : **(4×8=32)**

13) Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by Charpit's method.

14) Reduce $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ to canonical form.

15) Solve the equation $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

i) $u(0, t) = 0$ and $u(1, t) = 0$ for $t \geq 0$

ii) $u(x, 0) = x^2 - x$, $0 \leq x \leq 1$.

16) Solve $y''(t) + 2y'(t) + y(t) = te^{-t}$ given that $y(0) = 1$ and $y'(0) = -2$ using Laplace transforms.

17) Find the Fourier series of the function $f(x) = x + x^2$ in $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

18) Define the Fourier transform and find Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \leq 1 \\ 0, & \text{if } |x| > 1 \end{cases}.$$
