

5

CB – 168

43

IV Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2021-22 and Onwards) (Repeaters)
MATHEMATICS – IV

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** Parts.

PART – A

1. Answer **any five** questions :

(5×2=10)

- Define a normal subgroup.
- Define homomorphism of groups.
- Obtain half range sine series $f(x) = 2x - 1$ in the interval $(0, 1)$.
- Define Fourier series.
- Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$.
- Evaluate $\lim_{x \rightarrow 0} x^x$.
- Solve $y'' - 6y' + 13y = 0$.
- Find the particular integral of $y'' - 6y' + 9y = 3e^{-4x}$.



PART – B

2. Answer **any two** questions :

(2×5=10)

- Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H \forall g \in G, h \in H$.
- Prove that the product of any two normal subgroups of a group is again a normal subgroup of the same.
- State and prove fundamental theorem of Homomorphism.

PART – C

3. Answer **any two** questions :

(2×5=10)

- Obtain the Fourier expansion of the function $f(x) = e^x$ in $(-\pi, \pi)$.
- Obtain the Fourier half range cosine series for the function $f(x) = \sin x$ in $(0, \pi)$.
- Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

P.T.O.



PART - D

4. Answer **any three** questions :

(3×5=15)

- a) Discuss the continuity of the functions $f(x) = \begin{cases} 1+x & \text{for } x \leq 2 \\ 5-x & \text{for } x > 2 \end{cases}$ at $x = 2$.
- b) State and prove Rolle's theorem.
- c) Expand $\tan x$ upto the term containing x^5 using Maclaurin's expansion.
- d) Find the extreme values of $f(x, y) = 2x^2 - xy + y^2 + 7x$.
- e) Obtain the Taylor's expansion of $f(x, y) = x^2y + 3y - 4$ in power of $(x - 1)$ and $(y + 2)$.

PART - E

5. Answer **any three** questions :

(3×5=15)

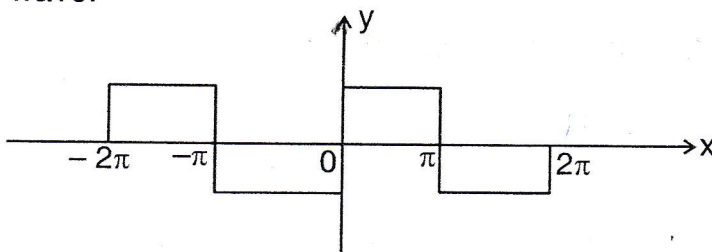
- a) Solve : $(D^2 - 5D + 6)y = e^{4x}$.
- b) Solve : $x^2y'' - xy' + 2y = x \log x$.
- c) Solve : $y'' + 2y' + 4y = e^x \sin x$.
- d) Solve : $xy'' + (2x^2 - 1)y' - 24x^3y = 4x^3 \sin x^2$ ($x > 0$) by using the transformation $z = x^2$.
- e) Solve by the method of variation of parameter $y'' + a^2y = \sec ax$.

PART - F

6. Answer **any two** questions :

(2×5=10)

- a) Sketch the graph of triangular wave and explain it.
- b) Obtain the Fourier series expansion of the following anti symmetrical square wave.



- c) Sketch the graph of sawtooth wave and explain it.