



CB – 169

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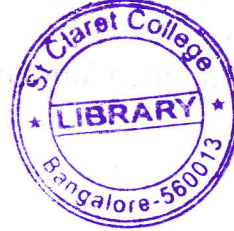
IV Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2015 – 16 and Onwards) (Repeaters)
Paper – IV : MATHEMATICS

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.

PART – A



Answer any five questions.

(5×2=10)

1. a) Define Homomorphism.
- b) If $f : G \rightarrow G'$ is a homomorphism, then prove that $f(e) = e'$ where e and e' are the identity element of G and G' respectively.
- c) Write the formula for b_n of fourier sine series expansion.
- d) Find the critical points for the function $f(x, y) = 2x^2 - xy + y^2 + 7x$.
- e) Find $L^{-1} \left\{ \frac{1}{3s^2 + 16} \right\}$.
- f) Find $L\{e^{2t} \cdot \sin 5t\}$
- g) Solve $(D^2 - 7D + 12) y = 0$.
- h) Find the particular integral of $(D^2 - 3D + 2) y = e^{5x}$.

PART – B

Answer one full question.

(1×15=15)

2. a) Prove that a subgroup H of a group G is normal subgroup if and only if $gHg^{-1} = H \quad \forall g \in G$.
- b) Prove that centre of a group G is normal subgroup of G .
- c) If $f : G \rightarrow G'$ is a homomorphism, then prove that set $f(G) = \{f(g) \mid g \in G\}$ is a subgroup of G' .

OR

P.T.O.



3. a) Prove that intersection of two normal subgroups of a group is a normal subgroup.
- b) Let $f : G \rightarrow G'$ be a homomorphism from the group G into G' with Kernel K then f is one-one if and only if $K = \{e\}$ where e is the identity element of G .
- c) State and prove fundamental theorem of homomorphism of group G .

PART – C

Answer **two full** questions.

(2×15=30)

4. a) Obtain the Fourier series for the function $f(x) = x^2$ over the interval $(-\pi, \pi)$.
- b) Find the half range sine series of $f(x) = (x - 1)^2$ in the interval $(0, 1)$.
- c) Expand $e^x \log(1 + y)$ in powers of x and y by Taylor series upto the third degree terms.

OR

5. a) Find the Fourier series for the function

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{in } -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & \text{in } 0 < x < \pi \end{cases}$$

- b) Find the extreme values of $f(x, y) = 2x^3 - xy + y^2 + 7x$.
- c) A rectangular box, open at the top, is to have a volume 32 cubic units. Find the dimensions so that total surface is minimum.
6. a) Find $L\{\sin 2t \cdot \sin 3t\}$.
- b) Find $L\{\cosh(2t) \cdot \cos 2t\}$.
- c) Find $L^{-1}\left\{\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}\right\}$.

OR



7. a) Find $L \left\{ \frac{\cos at - \cos bt}{t} \right\}$.

b) Find $L^{-1} \left\{ \log \left[\frac{s^2 + 1}{s(s + 1)} \right] \right\}$.

c) Verify convolution theorem for the function $f(t) = \sin t, g(t) = e^{-t}$.

PART – D

Answer **one full** question.

(1×15=15)

8. a) Solve : $(D^2 - 5D + 6)y = \sin 2x$.

b) Solve : $(D^2 - D - 6)y = x$.

c) Solve : $4x^2y'' + 4xy' - y = 4x^2$.

OR

9. a) Solve : $\frac{d^2y}{dx^2} + y = 5x^2e^x$.

b) Solve : $\frac{dx}{dt} = 3x - y, \frac{dy}{dt} = x + y$.

c) Solve : $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters.
