CB – 169

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IV Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2015 – 16 and Onwards) (Repeaters)
Paper – IV : MATHEMATICS

Time : 3 Hours   Max. Marks : 70

***Instruction** : Answer **all** Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Define Homomorphism.

   b) If $f : G \to G'$ is a homomorphism, then prove that $f(e) = e'$ where e and e′ are the identity element of G and G′ respectively.

   c) Write the formula for b_n of fourier sine series expansion.

   d) Find the critical points for the function $f(x, y) = 2x^2 - xy + y^2 + 7x$.

   e) Find $L^{-1}\left\{\dfrac{1}{3s^2+16}\right\}$.

   f) Find $L\{e^{2t} \cdot \sin 5t\}$

   g) Solve $(D^2 - 7D + 12)\, y = 0$.

   h) Find the particular integral of $(D^2 - 3D + 2)\, y = e^{5x}$.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) Prove that a subgroup H of a group G is normal subgroup if and only if $gHg^{-1} = H \ \forall g \in G$.

   b) Prove that centre of a group G is normal subgroup of G.

   c) If $f : G \to G'$ is a homomorphism, then prove that set $f(G) = \{f(g) \mid g \in G\}$ is a subgroup of G′.

OR

P.T.O.

3. a) Prove that intersection of two normal subgroups of a group is a normal subgroup.

   b) Let $f : G \to G'$ be a homomorphism from the group G into G′ with Kernal K then f is one-one if and only if $K = \{e\}$ where e is the identity element of G.

   c) State and prove fundamental theorem of homomorphism of group G.

PART – C

Answer **two full** questions. **(2×15=30)**

4. a) Obtain the Fourier series for the function $f(x) = x^2$ over the interval $(-\pi, \pi)$.

   b) Find the half range sine series of $f(x) = (x - 1)^2$ in the interval (0, 1).

   c) Expand $e^x \log(1 + y)$ in powers of x and y by Taylor series upto the third degree terms.

OR

5. a) Find the Fourier series for the function

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi} & \text{in} \quad -\pi < x < 0 \\ 1-\dfrac{2x}{\pi} & \text{in} \quad 0 < x < \pi \end{cases}.$$

   b) Find the extreme values of $f(x, y) = 2x^3 - xy + y^2 + 7x$.

   c) A rectangular box, open at the top, is to have a volume 32 cubic units. Find the dimensions so that total surface is minimum.

6. a) Find $L\{\sin 2t \cdot \sin 3t\}$.

   b) Find $L\{\cosh(2t) \cdot \cos 2t\}$.

   c) Find $L^{-1}\left\{\dfrac{5s+3}{(s-1)(s^2+2s+5)}\right\}$.

OR

7. a) Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$.

b) Find $L^{-1}\left\{\log\left[\frac{s^2+1}{s(s+1)}\right]\right\}$.

c) Verify convolution theorem for the function $f(t) = \sin t$, $g(t) = e^{-t}$.

PART – D

Answer **one full** question. **(1×15=15)**

8. a) Solve : $(D^2 - 5D + 6)y = \sin 2x$.

b) Solve : $(D^2 - D - 6)y = x$.

c) Solve : $4x^2y'' + 4xy' - y = 4x^2$.

OR

9. a) Solve : $\frac{d^2y}{dx^2} + y = 5x^2e^x$.

b) Solve : $\frac{dx}{dt} = 3x - y, \frac{dy}{dt} = x + y$.

c) Solve : $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters.
