CB – 170

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VI Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2022 – 23 and Onwards) (Freshers)
MATHEMATICS (Paper – VII)

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

I. Answer **any five** questions. **(5×2=10)**

1) Define vector space over a field F.

2) Prove that the set $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is linearly independent in $V_3(R)$.

3) Prove that $T : V_1(R) \to V_3(R)$ defined by $T(x) = (x, x^2, x^3)$ is not a linear transformation.

4) Write scale factors in spherical co-ordinate system.

5) In spherical system P.T. $\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi$.

6) Define coordinate surface.

7) Solve $p^2 + q^2 = 1$.

8) Solve $(D^2 - 2DD' + D'^2) Z = 0$.

PART – B

II. Answer **any three** of the following. **(3×5=15)**

9) Prove that a non empty subset W of a vector space V(F) is a subspace of V(F) if and only if $\forall\, \alpha, \beta \in W$ and any $a, b \in F$, $a\alpha + b\beta \in W$.

10) Find the dimension and basis of the subspace spanned by the vectors (2, –3, 1), (3, 0, 1), (0, 2, 1) and (1, 1, 1) in $V_3(R)$.

11) Verify whether $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (3x + 2y, 3x - 4y)$ is a linear transformation or not.

12) Find the matrix of linear transformation.

$T : R^2 \to R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to the basis $B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, 3), (2, 5)\}$.

13) State and prove Rank and Nullity theorem.

P.T.O.

PART – C

III. Answer **any three** of the following. **(3×5=15)**

14) Express the vector $\vec{f} = 2yi - zj + 3xk$ in cylindrical co-ordinate and find f_ρ, f_ϕ, f_z.

15) Express the vector $\vec{f} = zi - 2xj + yk$ in spherical co-ordinates and find f_r, f_θ, f_ϕ.

16) Show that the spherical co-ordinate system is orthogonal curvilinear co-ordinate system and also prove that f_r, f_θ, f_ϕ form a right handed basis.

17) Show that the cylindrical co-ordinate system is orthogonal co-ordinate system.

18) Express the base vectors e_1, e_2, e_3 in terms of i, j, k.

PART – D

IV. Answer **any four** of the following. **(4×5=20)**

19) Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

20) Form the partial differential equation by eliminating the arbitrary function $Z = f(x + ay) + g(x - ay)$.

21) Solve $x^2p^2 + y^2q^2 = z^2$.

22) Solve by Charpit's method $px + qy = pq$.

23) Solve $(D^2 - 5DD' + 4D'^2) Z = \cos(2x + 3y)$.

24) A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in a position given by $y = y_0 \mathrm{Sin}^3\left(\frac{nx}{l}\right)$. If it is released from the initial position, find the displacement y(x, t).

PART – E

V. Answer **any two** of the following. **(2×5=10)**

25) A scientist has found two solutions to a homogeneous system of 40 equations in 42 variables. The two solutions are not multiples, and all other solutions can be constructed by adding together appropriate multiples of these two solutions. Can the scientists be certain that an associated non homogeneous system (with the same co-efficients) has a solution ?

26) An infinitely long rectangular uniform plate with breadth π is bounded by two parallel edges maintained at 0°C. Base of the plate at a temperature u_0 at all points. Determine the temperature at any point of the plate in the steady state.

27) A rod of length 'l' with insulated side is initially at a uniform temperature x_0. It's ends are suddenly cooled at 0°C and are kept at the temperature. Find the temperature formula U(x, t).