52

CB – 171

VI Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (F+R) (2016 – 17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) In a Vector Space V over the field F, show that $(-a)\cdot\alpha = -(a\cdot\alpha)\ \forall\ a \in F$, $\alpha \in V$.

   b) Show that $W = \{(0, 0, z) | z \in R\}$ is a subspace of $V_3(R)$.

   c) Show that the vectors $\alpha_1 = (1, 1, 0)$, $\alpha_2 = (1, 1, 0)$ and $\alpha_3 = (1, 0, 0)$ are linearly independent.

   d) If $T : V_2 \to V_2$ defined by $T(x, y) = (x + y, y)$, then show that T is a linear transformation.

   e) Write scalar factors in cylindrical co-ordinate system.

   f) Solve $\frac{dx}{zx} = \frac{dy}{yz} = \frac{dz}{xy}$.

   g) Form a partial differential equation by eliminating arbitrary constants from $z = (x - a)^2 + (y - b)^2$.

   h) Solve $p^2 + q^2 = 1$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Prove that the intersection of any two subspaces of a vector space V(F) is also a subspace of V. But the union of two subspaces of vector space V(F) need not to be subspace of V. Justify.

   b) State and prove the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V.

OR

3. a) Show that the vector (3, –7, 6) is a linear combination of the vectors (1, –3, 2), (2, 4, 1) and (1, 1, 1).

   b) In a n-dimensional vector space V(F), prove that

      i) any (n + 1) elements of V are linearly dependent.

      ii) No set of (n – 1) elements can span V.

4. a) State and prove rank-nullity theorem.

   b) Show that the linear transformation $T : R^3 \to R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ is non-singular where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

OR

5. a) Find the linear transformation $T : R^2 \to R^2$ such that T(1, 1) = (0, 1, 2) and T(–1, 1) = (2, 1, 0).

   b) Find the matrix of linear transformation $T : V_3(R) \to V_2(R)$ defined by T(x, y, z) = (x + y, y + z) relative to basis $B_1 = \{(1, 1, 1), (1, 0, 0), (1, 1, 0)\}$ and $B_2 = \{e_1, e_2\}$ of $V_3(R)$ and $V_2(R)$ respectively.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Verify the condition of integrability and solve 2yzdx + zxdy – xy(1 + z)dz = 0.

   b) Solve (y – z)p + (z – x)q = x – y.

OR

7. a) Show that cylindrical system is orthogonal curvilinear co-ordinate system.

   b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical co-ordinates and find f_r, f_θ, f_ϕ.

8. a) Solve $\dfrac{dx}{mz - ny} = \dfrac{dy}{nx - lz} = \dfrac{dz}{ly - mx}$.

   b) Solve $\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{z^2 - xy}$.

OR

9. a) Express $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical co-ordinates system and find f_ρ, f_ϕ, f_z.

b) Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in the form of spherical polar co-ordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Form partial differential equation by eliminating arbitrary function from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

b) Solve $x(1 + y)p = y(1 + x)q$.

OR

11. a) Solve $(D^2 + DD' - 6(D')^2)z = \cos(2x + y)$

b) Solve $z^2(p^2z^2 + q^2) = 1$.

12. a) Solve $px + qy = pq$ by Charpit's method.

b) Solve $(D^2 - DD' - 6(D')^2)z = xy$.

OR

13. a) Solve $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ given that

i) $u(0, t) = 0$, $u(l, t) = 0$ for all $t \geq 0$ and

ii) $u(x, 0) = f(x)$, $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = \phi(x)$ for $0 < x < l$.

b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ given that

i) $u(0, t) = 0$, $u(1, t) = 0$

ii) $u(x, 0) = x^2 - x$, $0 \leq x \leq 1$.