53

CB – 173

VI Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (2016 – 17 and Onwards) (Repeaters)
MATHEMATICS – VIII

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

   a) Evaluate $\lim_{z \to 1+2i} (z^2 + 1)$.

   b) Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic.

   c) Find the locus of z satisfying $|z - 1| \leq 2$.

   d) State Liouville's theorem.

   e) Verify Cauchy-Reimann equations for f(z) = sinz.

   f) State Fundamental theorem of algebra.

   g) Find the real root of the equation $x^3 - 4x + 9 = 0$ in one step by bisection method.

   h) Using Newton-Raphson method, find the real root of $x^3 - 2x - 5 = 0$ in one iteration only.

PART – B

Answer **four full** questions. **(4×10=40)**

2. a) Show that the locus of $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ is a line through the origin.

   b) State and prove necessary conditions for a function f(z) = u + iv to be analytic.

OR

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3. a) Evaluate $\lim_{z \to 2e^{i\pi/3}} \left(\frac{z^3 + 8}{z^4 + 4z^2 + 16} \right)$.

   b) Show that f(z) = log z is analytic and find f′(z).

4. a) If f(z) = u + iv is analytic, show that $\left[\frac{\partial}{\partial x} | f(z) | \right]^2 + \left[\frac{\partial}{\partial y} | f(z) | \right]^2 = | f'(z) |^2$.

   b) Prove that $u = y^3 - 3x^2y$ is a harmonic function and find its harmonic conjugate.

OR

5. a) Find the orthogonal trajectories of the family of curves $2e^{-x}\sin y + x^2 - y^2 = C$.

   b) If f(z) = u + iv and $u - v = e^x(\cos y - \sin y)$, find f(z) in terms of z.

6. a) Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$.

   b) State and prove Cauchy's integral formula.

OR

7. a) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C : |z| = 3.

   b) Evaluate $\int_C \frac{dz}{z^2 - 4}$ over C : |z + 2| = 1.

8. a) Find the bilinear transformation which map the points z = 1, i, –1 into w = 2, i, –2.

   b) Show that the transformation $w = \frac{i - z}{i + z}$ maps the x-axis of the z-plane onto a circle |w| = 1 and the points in the half plane y > 0 on the points |w| < 1.

OR

9. a) Prove that the Bilinear transformation preserves the cross ratio of four points.

   b) Discuss the transformation w = sinz.

PART – C

Answer **two full** questions. **(2×10=20)**

10. a) Find the root of the equation $f(x) = x^3 - 4x + 1$ by Regula-Falsi method upto three decimal places.

    b) Using Newton-Raphson method, find the real root of equation $x^4 - x - 10 = 0$ which is near to $x = 2$ correct to 3 decimal places.

OR

11. a) Solve by Gauss-Jacobi method $10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$.

    b) Find the largest eigen value of the matrix $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by power method.

12. a) Find y at x = 0.1 correct to 4 decimal places, given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ applying Taylor's series method upto fourth degree term.

    b) Using Euler's method, solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ for $x = 0.0(0.2)1.0$.

OR

13. a) Using modified Euler's method, find y(0.1) given $\frac{dy}{dx} = x^2 + 1$, $y(0) = 1$.

    b) Using Runge-Kutta method find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$ with $y(0) = 1$ taking $h = 0.2$.