Roll No:
Date: __/__/____

St. Claret College

Autonomous, Bengaluru

UG END SEMESTER EXAMINATION-DECEMBER 2024
BSC- I SEMESTER
MT 124: MATHEMATICS- I

TIME: 3 hours II **MAX. MARKS: 80**

This paper contains FOUR printed pages and FOUR parts

Instructions:
1. **Verify and ensure that the question paper is completely printed.**
2. **Any discrepancies or questions about the exam paper must be reported to the HOD within 1 hour after the examination.**
3. **Students must check the course title and course code before answering the question paper.**

PART-A

Answer ALL questions. Each answer carries ONE mark. **[1 x 10 = 10]**

1. What are the eigenvalues of an identity matrix of order 6?
   a) 6
   b) 1
   c) -1
   d) 3
2. Rank of a null matrix is
   a) 1
   b) -1
   c) Not defined
   d) 0
3. If r = n, then the system of homogeneous linear equations will have
   a) Only trivial solution
   b) Non-trivial solution
   c) Unique solution
   d) Infinitely many solutions

4. What is the n^{th} derivative of e^{ax}, where a is a constant?

   a) $a^n e^{ax}$
   b) e^{ax}
   c) $n!\, e^{ax}$
   d) $(-1)e^{ax}$

5. If f(x) = $cot\ x$, what is $f''(x)$?

   a) $-\text{cosec}^2 x$
   b) $2cosec^2 x\ cot\ x$
   c) $-2cosec^2 x$
   d) $cosec^2 x$

6. Evaluation of $\int_0^{\frac{\pi}{2}} cos^5\ x\ dx\ is$

   a) $\frac{8}{15}$
   b) $\frac{88}{25}$
   c) $\frac{38}{15}$
   d) $\frac{28}{80}$

7. The area of the Lemniscate $r^2 = a^2\ cos2\theta$ is

   a) a^2
   b) a^3
   c) a^4
   d) a^6

8. The equation $x^2 + y^2 + z^2 = 49$ represents a sphere with

   a) Radius 49 and center at (0, 0, 0)
   b) Radius 7 and center at (0, 0, 0)
   c) Radius 7 and center at (49, 0, 0)
   d) Radius 7 and center at (1, 1, 1)

9. For what value of k the two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10\,y - 4z - 8 = 0$ cuts orthogonally?

   a) $k = 1$
   b) $k = 4$
   c) $k = 6$
   d) $k = 2$

10. The equation of a right circular cone in its standard form is

   a) $x^2 + y^2 = z^2\ tan^2\alpha$
   b) $x^2 + y^2 + z^2 = tan^2\alpha$
   c) $x^2 + y^2 = z^2\ tan^3\alpha$
   d) $x^2 + y^2 - z^2 = tan^2\alpha$

PART-B

Answer any TEN questions. Each answer carries TWO marks. **[2 x 10 = 20]**

11. Find the rank of the matrix A = $\begin{bmatrix} 4 & 3 & 2 & 2 \\ 5 & 1 & 3 & 4 \end{bmatrix}$.
12. Define symmetric matrix with an example.
13. State the Cayley Hamilton Theorem.
14. Find the n^{th} derivative of *cos (1-4x)*.
15. Find the n^{th} derivative of $cos^4 x$.
16. Find the total differential of $f(x, y, z) = x + y + z + xyz$.
17. Evaluate $\int_0^{\frac{\pi}{2}} sin^6\, x\, dx$.
18. Evaluate $\int_0^{\frac{\pi}{2}} sin^5\, x\, cos^6\, x\, dx$.
19. Evaluate $\int_0^{\pi} \frac{1-cos\theta}{1+cos\theta}\, sin^2\theta\, d\theta$.
20. Find the equation of the sphere whose centre is (2, -3, 4) and radius is 5.
21. Show that the spheres $x^2 + y^2 + z^2 + 6y + 14z + 28 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 24 = 0$ cut orthogonally.
22. Find the equation of the right circular cone whose vertex is (2, -3, 5), axis makes equal angles with the coordinate axes and with semi vertical angle $cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$.

PART-C

Answer any FOUR questions. Each answer carries FIVE marks. **[5 x 4 = 20]**

23. Show that the system of equations $x+ y+ z=6;\ x+ 2y+ 3z=14;\ x+ 4y+ 7z=30$ is consistent and solve them.
24. Find the n^{th} derivative of $\frac{1}{6x^2-5x+1}$
25. Verify Euler's theorem for the function $u = \frac{x\,(x^3-y^3)}{(x^3+y^3)}$
26. Find the length of the arc of the parabola $y^2 = 4ax$ which is intercepted between the points of intersection with the line $y = 2x$.
27. Find the area bounded by the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
28. Find the equation of a right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, -3, 6.

PART-D

Answer any THREE questions. Each answer carries TEN marks. **[10 x 3 = 30]**

29. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$.

30. Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$ and hence find its inverse.

31. If $u = x^2\text{-}2y$ and $v = x+y$, find $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$ and verify that $JJ' = 1$.

32. a) Find the area of the surface generated by revolving about the y axis the curve $x= y^3$ from y = 0 to y = 2. (5 marks)

    b) Find the volume of the solid obtained by revolving the cardioid r = a (1+ cosθ) about the initial line. (5 marks)

33. a) Find the equation of the tangent plane to the sphere $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at (1, 2, 3). (4 marks)

    b) Find the right circular cylinder generated by revolving the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$ about the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+5}{-1}$. (6 marks)

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