UG – 172

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VI Semester B.A./B.Sc. Examination, Sept./Oct. 2022
(Semester Scheme)
(CBCS) (F+R) (2016-17 and Onwards)
MATHEMATICS – VIII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

   a) Evaluate $\lim\limits_{z \to 1+2i} (z^2 + 1)$.

   b) Find the locus of z satisfying $|z - i| \leq 3$.

   c) Show that $v = 3x^2y - y^3$ is harmonic.

   d) State Liouville's theorem.

   e) Define bilinear transformation.

   f) Verify Cauchy-Riemann equations for f(z) = sinx coshy + i cosx sinhy.

   g) Find the real root of the equation $x^3 - 4x + 9 = 0$ in one step by bisection method.

   h) Write Euler's modified formula.

PART – B

Answer **four full** questions. **(4×10=40)**

2. a) Show that $|z + i|^2 - |z - i|^2 = 2$ represents a real axis.

   b) State and prove necessary conditions for a function f(z) = u + iv to be analytic.

OR

3. a) Evaluate $\lim\limits_{z \to \frac{i}{2}} \left[\dfrac{(2z^3 - 3)(4z + i)}{(iz - 1)^2}\right]$.

   b) Show that f(z) = log z is analytic and find f′(z).

P.T.O.

4. a) Find the analytic function f(z) = u + iv given its real part $u = \left(r + \frac{1}{r}\right) \cos\theta$.

b) Show that u = e^x siny + $x^2 - y^2$ is harmonic and find its harmonic conjugate.

OR

5. a) If f(z) is analytic, show that $\left(\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2}\right) | f(z) |^2 = 4 | f'(z) |^2$.

b) Find the orthogonal trajectory of the family of curves $x^2 - y^2 - x = c$.

6. a) If f(z) is analytic with in and on a closed curve C of a simply connected region and z = z_0 is an interior point of C, prove that $\frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)} dz = f(z_0)$.

b) Evaluate $\int_C \frac{z}{(z^2 + 1)(z^2 - 9)} dz$, where C is the circle |z| = 2.

OR

7. a) State and prove fundamental theorem of algebra.

b) If C is the circle with centre 'a' and radius 'r' then show that

i) $\int_C \frac{1}{(z - a)} dz = 2\pi i$

ii) $\oint_C (z - a)^n dz = 0$, if $n \neq -1$.

8. a) Discuss the transformation $\omega = \sin z$.

b) Show that the bilinear transformation preserves the cross ratio of four points.

OR

9. a) Find the bilinear transformation which maps 0, i, ∞ onto 1, –i, –1.

b) Show that $W = \frac{2z + 3}{z - 4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line 4u + 3 = 0.

PART – C

Answer **two full** questions. **(2×10=20)**

10 a) Find the real root of $xe^x - 2 = 0$ by using Regula – Falsi method correct to three decimal places in (0,1).

b) Using Newton-Raphson's method, find the cube root of 37.

OR

11 a) Solve $10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 14$ by Jacobi iteration method.

b) By using power method, find the largest eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ given $x_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ in five steps.

12 a) Using Taylor's series method, find the solution of $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ at $x = 0.2$.

b) Solve $\frac{dy}{dx} = x + y$ with $x_0 = 0$, $y_0 = 1$ for $x = 0(0.05)\,0.05$ using Euler's modified method.

OR

13 a) Find the approximate solution at $x = 1.2$ of the equation $\frac{dy}{dx} = xy$, given $y(1) = 2$ by Runge Kutta method by taking $h = 0.2$.

b) Solve $\frac{dy}{dx} = 1 + \frac{y}{x}$ with $y(1) = 2$, find $y(1.4)$ taking $h = 0.4$ by Euler's modified method.