CS – 163

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V Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2022-23 and Onwards) (Fresh)
Paper – V : MATHEMATICS

Time : 3 Hours Max. Marks : 70

Instruction *: Answer **all** Parts.*

PART – A

I. Answer **any five** questions : **(5×2=10)**

1) In a ring (R, + ,·), prove that $a.(b-c) = a.b - a.c \ \forall \ a, b, c \in R$.

2) Define left and right ideal of ring.

3) If F is a homomorphism of a ring R into R′ then prove that f(0) = 0′, where 0 and 0′ are the identity element of R and R′ respectively.

4) Write the Euler's equation when f is dependent of x.

5) Find the function y which makes the integral $I = \int_{x_1}^{x_2} [1 + xy' + (y')^2]dx$.

6) Prove that $E\nabla = \nabla E = \Delta$.

7) Write the Lagrange's inverse interpolation formula.

8) Write the Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule formula.

PART – B

II. Answer **any three** questions. **(3×5=15)**

9) Prove that intersection of any two subrings of a ring are subring. Give an example to show that union of two subrings of a ring need not be a subring.

10) Prove that the set R = {0, 1, 2, 3, 4, 5} is a commutative ring w.r.t. addition and multiplication modulo 6.

11) Prove that the set of all matrices of the form $M = \left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : a, b \in R\right\}$ is a non- commutative ring without unity w.r.t. addition and multiplication of matrices.

P.T.O.

12) Prove that $(Z_5, +_5, \times_5)$ is an integral domain w.r.t. addition and multiplication modulo 5.

13) State and prove fundamental theorem of homomorphism.

PART – C

III. Answer **any three** questions. **(3×5=15)**

14) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

15) Show that the extremal of the functional $\int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is expressible in the form $y = ae^{bx}$.

16) Define Geodesic. Prove that geodesic on plane is a straight line.

17) If a cable hangs freely under gravity from the fixed points, then show that the shape of the curve is catenary.

18) Find the extremal of the functional $I = \int_0^{\pi} \left((y')^2 - y^2\right)dx$ under the conditions $y(0) = 0$, $y(\pi) = 1$ and subjected to the constraint $\int_0^{\pi} ydx = 1$.

PART – D

IV. Answer **any four** questions. **(4×5=20)**

19) Find the cubic polynomial which takes the following data.

x	0	1	2	3
f(x)	1	2	1	10

20) Apply Newton backward interpolation formula find f(84) from the following data.

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

21) Express $3x^3 - 4x^2 + 3x - 11$ in factorial notation and also find their successive differences.

22) Use the method of separation of symbols to prove that
$u_0 + u_1 + u_2 + \dots + u_n = {}^{n+1}c_1 u_0 + {}^{n+1}c_2 \Delta u_0 + {}^{n+1}c_3 \Delta^2 u_0 + \dots + \Delta^n u_0$.

23) Using Lagrange's interpolation formula find f(10) from the following data.

x	5	6	9	11
y = f(x)	12	13	14	16

24) Evaluate $\int_1^5 \log_{10} x \, dx$ by using trapezoidal rule, divide [1, 5] into eight equal parts.

PART – E

V. Answer **any two** questions. **(2×5=10)**

25) Find the velocity and acceleration at time t = 1 from the following data.

t	1	2	3	4	5	6
f(t)	1	8	27	64	125	216

26) The specific gravity of zinc sulphate solution of various concentration at 15°C is given in the table. Obtain the specific gravity of 15.8% at 15°C.

Conce.	10	12	14	16	18	20	22
Spec. gra.	1.059	1.073	1.085	1.097	1.110	1.124	1.137

27) Find the path in which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity.