CS – 165

71

V Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2022 – 23 and Onwards) (Fresh)
MATHEMATICS
Paper 6(A) : Elective – I

Time : 3 Hours Max. Marks : 70

Instruction : *Answer* ***all*** *Parts.*

PART – A

I. Answer **any five** questions. **(5×2=10)**

1) If $\phi = x^2 + y^2 + 4z^2$, find $\nabla^2 \phi$.

2) Show that the vector $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.

3) Show that curl (grad φ) = 0.

4) Evaluate $\int_c (x^2 - y)dx + (y^2 + x)dy$, where c is the curve given by $x = t,\ y = t^2 + 1,\ 0 \le t \le 1$.

5) Evaluate $\int_0^2 \int_1^2 (x^2 + y^2)dx\,dy$.

6) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz$.

7) Using Stoke's theorem prove that $\operatorname{div}(\operatorname{curl}\vec{F}) = 0$.

8) If V is the volume of a region bounded by a closed surface s, show that $\iint_s \vec{r}.\hat{n}\,ds = 3V$.

PART – B

II. Answer **any four** questions. **(4×5=20)**

9) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.

10) If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{F}$ is irrotational, then find φ such that $\vec{F} = \nabla\varphi$.

P.T.O.

11) Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point (1, –1, 2).

12) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\operatorname{div}\left(\frac{\vec{r}}{r^2}\right) = \frac{1}{r^2}$.

13) Show that $\vec{f} = 2xyz\,\hat{i} + x^2z\,\hat{j} + x^2y\,\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{f} = \nabla\phi$.

14) Divergence of a vector product for any vector fields $\vec{f} \times \vec{g}$, then prove that $\nabla \cdot (\vec{f} \times \vec{g}) = \vec{g} \cdot (\nabla \times \vec{f}) - \vec{f}(\nabla \times \vec{g})$.

PART – C

III. Answer **any five** questions. **(5×5=25)**

15) Evaluate $\int_c (3x + y)dx + (2y - x)dy$ along the line joining the points (0, 1) and (3, 10).

16) Evaluate $\int_c (x + y + z)dz$, where c is the line joining the points (1, 2, 3) and (4, 5, 6).

17) Evaluate $\iint_R xy\,dx\,dy$, where R is the region bounded by the x-axis, the ordinates x = 2a and the parabola $x^2 = 4ay$, a > 0.

18) Evaluate $\iint_R \frac{x^2y^2}{x^2 + y^2}\,dx\,dy$, where R is the annular region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$.

19) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2}\,dxdy$.

20) Evaluate $\int_1^2 \int_0^{1-x} \int_0^{1-x-y} \frac{dxdydz}{(x + y + z)^3}$.

21) Prove that $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}} = \frac{\pi^2 a^2}{8}$ by changing to spherical polar co-ordinates.

22) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

PART – D

IV. Answer **any three** questions. **(3×5=15)**

23) State and prove Green's theorem.

24) Verify Green's theorem in a plane for $\int_c (3x^2 - 8y^2)dx + 2y(2-3x)dy$, where c is the rectangle enclosed by the lines x = 0, y = 0 and x + y = 1.

25) Using Gauss divergence theorem, evaluate $\iint_s \vec{F}.\hat{n}ds$, where $\vec{F} = 2xy\,\hat{i} + yz^2\,\hat{j} + xz\,\hat{k}$ and s is the total surface of the rectangular parallelopiped bounded by the points x = 0, y = 0, z = 0 and x = 1, y = 2, z = 3.

26) State and prove Stoke's theorem.

27) Evaluate by Stoke's theorem $\oint_c yzdx + zxdy + xydz$, where c is the curve $x^2 + y^2 = 1$, $z = y^2$.