# I Semester B.C.A. Degree Examination, March 2023 <br> (Y2K14) (CBCS) (R) <br> COMPUTER SCIENCE 

## BCA 105T : Discrete Mathematics

## Time : 3 Hours

Instruction : Answer all Sections.

> SECTION - A

I. Answer any ten of the following.

1) If $A=\{a, b\}, B=\{c, d, e\}$ find $A \times B$.
2) Define an equivalence relation on a set.
3) Define scalar matrix with an example.
4) Define Tautology.
5) If $A=\left[\begin{array}{cc}2 & -1 \\ 4 & 0\end{array}\right], B=\left[\begin{array}{cc}5 & 2 \\ -3 & 2\end{array}\right]$ find $3 A-2 B$.
6) If $\log _{2} 64=x$, then find $x$.
7) Define permutation and combination.
8) Define group.
9) Define onto function.
10) If $\vec{a}=2 \hat{i}+\hat{j}-3 \hat{k}$ and $\vec{b}=5 \hat{i}+\hat{j}+4 \hat{k}$, find $|\vec{a}+\vec{b}|$.
11) Find the distance between the points $A(3,-1)$ and $B(4,-2)$.
12) Find the slope of the line $3 x-2 y+5=0$.
SECTION - B
II. Answer any six of the following.
$(6 \times 5=30)$
13) If $A=\{1,2,3,4\}, B=\{3,4,5\}$ and $C=\{3,5,6,7\}$, then verify
$A \times(B \cup C)=\{A \times B\} \cup\{A \times C\}$
14) If $F: R \rightarrow R$ is defined by $f(x)=2 x+3$. Prove that ' $f$ ' is one-one and onto and hence find inverse of ' $f$ '.
15) Show that $\sim(p \rightarrow q) \leftrightarrow p \wedge \sim q$ is a tautology.
16) Prove that $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
17) Find the inverse of $A=\left[\begin{array}{ccc}3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5\end{array}\right]$.
18) Verify Cayley-Hamilton theorem for the matrix $A=\left[\begin{array}{cc}3 & 1 \\ -1 & 2\end{array}\right]$
19) Solve by Cramer's rule $3 x-y=13, x+3 y=-8$.
20) Verify Cayley-Hamilton theorem for the matrix $A=\left[\begin{array}{ll}3 & 2 \\ 4 & 5\end{array}\right]$. SECTION - C
III. Answer any six of the following.
21) If $\log \left(\frac{a+b}{3}\right)=\frac{1}{2}(\log a+\log b)$, then prove that $a^{2}+b^{2}=7 a b$.
22) In how many ways the letter of the word "EVALUATE" be arranged so that all vowels are together ?
23) Prove that the set $G=\{2,4,6,8\}$ is an abelian group under multiplication modulo 10.
24) Prove that the set $G=\{1,-1, i,-i\}$ is a group under multiplication.
25) If ${ }^{2 n} C_{3}:{ }^{n} C_{2}=44: 3$, find $n$.
26) Find the value of ' $\lambda$ ' for which the vectors $\vec{a}=3 \hat{i}+\hat{j}-2 \hat{k}$ and $\vec{b}=\hat{i}+\lambda \hat{j}-3 \hat{k}$ are perpendicular to each others.
27) Show that the points $A(1,2,3), B(2,3,1)$ and $C(3,1,2)$ are vertices of an equilateral triangle.
28) If the vectors $4 \hat{i}+11 \hat{j}+m \hat{k}, 7 \hat{i}+2 \hat{j}+6 \hat{k}$ and $\hat{i}+5 \hat{j}+4 \hat{k}$ are coplanar, then find ' $m$ '.

## SECTION - D

IV. Answer any four of the following.
$(4 \times 5=20)$
29) Prove that the points $(6,4),(7,-2),(5,1),(4,7)$ form vertices of a parallelogram.
30) The three vertices of a parallelogram taken in order are (8, 5), ( $-7,-5$ ) and $(-5,5)$. Find the co-ordinate of the fourth vertex.
31) Find the equation of line passing through $(1,-2)$ and parallel to the line $2 x+3 y+4=0$
32) Find the root of the perpendicular drawn from $(-3,5)$ on the line $x-y-5=0$.
33) Show that the line $x-y+3=0,2 x-7 y+1=0, x-6 y-2=0$ are concurrent.
34) Find ' $k$ ' for which the lines $2 x-k y+1=0$ and $x+(k+1) y-1=0$ are perpendicular.

