



SS – 347

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**I Semester B.A./B.Sc. Examination, Nov./Dec. 2018**  
**(Semester Scheme) (2011-12 and Onwards) (N.S.)**  
**(Repeaters – Prior to 2014-15)**  
**MATHEMATICS – I**

Time : 3 Hours



Max. Marks : 100

**Instruction : Answer all questions.**

I. Answer **any fifteen** questions.

**(15×2=30)**

1) Reduce the matrix  $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$  to Echelon form.

2) State Cayley-Hamilton theorem.

3) Verify the equations  $x + 2y - z = 3$ ,  $3x - y + 2z = 1$ ,  $2x - 2y + 3z = 2$  for consistency.

4) Find the eigenvalue for the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .

5) Find the  $n^{\text{th}}$  derivative of  $\cos 2x$ .

6) Find the  $n^{\text{th}}$  derivative of  $e^{2x}$ .

7) If  $u = x \tan y + y \tan x$ , find  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$ .

8) State Euler's theorem for homogeneous functions.

9) Find  $\frac{du}{dt}$  if  $u = e^x \sin y$  where  $x = \log t$ ,  $y = t^2$ .

10) If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , find  $\frac{\partial(x.y)}{\partial(r.\theta)}$ .

11) Evaluate  $\int_0^{\pi/2} \cos^6 x \, dx$ .

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- 12) Using reduction formula, evaluate  $\int_0^{\pi/4} \tan^6 x \, dx$ .
- 13) Find the direction cosines of a line which makes angles  $90^\circ$ ,  $60^\circ$  and  $30^\circ$  with the co-ordinate axes.
- 14) Show that the lines whose direction ratios are  $(2, 3, 4)$  and  $(1, -2, 1)$  are at right angles.
- 15) Find the projection of the line segment AB on CD where  $A = (3, 4, 5)$ ,  $B = (4, 6, 3)$ ,  $C = (-1, 2, 4)$  and  $D = (1, 0, 5)$ .
- 16) Show that the planes  $2x - 4y + 3z + 5 = 0$  and  $10x + 11y + 8z - 17 = 0$  are perpendicular.
- 17) Find the angle between the planes  $x - y + z = 6$  and  $2x + 3y - 3z + 5 = 0$ .
- 18) Find the equation of the sphere whose centre at  $(2, -3, 4)$  and radius equal to 5 units.
- 19) If a right circular cone has three mutually perpendicular generators show that the semi vertical angle is  $\tan^{-1}\sqrt{2}$ .
- 20) Find k if the spheres  $x^2 + y^2 + z^2 + 6y + 2z + k = 0$  and  $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$  cut orthogonally.

II. Answer **any two** questions.

**(2×5=10)**

- 1) Reduce the given matrix to normal form and hence determine its rank

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

- 2) Using Cayley-Hamilton, find  $A^3$  if  $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ .

- 3) Verify the system of equations for consistency and if consistent solve them  $x + y - 2z = 5$ ,  $x - 2y + z = -2$  and  $-2x + y + z = 4$ .

- 4) Find the eigenvalues and the eigenvectors of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .



III. Answer **any four** questions.

(4×5=20)

- 1) Find the  $n^{\text{th}}$  derivative of  $\frac{x^4}{(x-1)(x-2)}$ .
- 2) If  $y = \sin(m \sin^{-1}x)$  prove that  $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$ .
- 3) If  $z = \sin(ax + y) + \cos(ax - y)$  prove that  $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$ .
- 4) State and prove Euler's theorem.
- 5) If  $u = \log \frac{x^4 + y^4}{x - y}$  show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ .
- 6) If  $u = x + y + z, v = y + z, w = z$ , find  $\frac{\partial(u.v.w)}{\partial(x.y.z)}$ .

IV. Answer **any two** questions.

(2×5=10)

- 1) Evaluate  $\int_0^{\pi} x \sin^7 x \, dx$ .
- 2) Obtain the reduction formula for  $\int \cos^n x \, dx$ .
- 3) Using Leibnitz's rule of differentiation under integral sign, evaluate  $\int_0^1 \frac{x^\alpha - 1}{\log x} \, dx$   
where  $\alpha$  is a parameter.

V. Answer **any four** questions.

(4×5=20)

- 1) Find the angles of the triangle ABC where  $A = (1, -2, -3), B = (2, -3, -1)$   
 $C = (3, -1, -2)$ .
- 2) Find the value of 'a' such that the points  $(3, 2, 1) (4, a, 5) (4, 2, -2)$  and  $(6, 5, -1)$  are coplanar.
- 3) Find the co-ordinates of the foot of the perpendicular drawn from the point  $(1, 2, 3)$  to the line  $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ .
- 4) Find the equation of the plane which contains the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{5}$   
and parallel to the line  $\frac{x}{4} = \frac{y}{1} = \frac{z}{2}$ .