CS – 159

I Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2020 – 21 and Onwards) (Repeaters)
MATHEMATICS (Paper – I)

Time : 3 Hours | Max. Marks : 70

***Instruction :** Answer **all** questions.*

PART – A (5×2=10)

I. Answer **any five** questions :

a) State Cayley-Hamilton theorem.

b) Find the eigenvalues of the matrix.

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

c) Find the n^{th} derivative of $\sin(3x - 1)$.

d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = e^{x/y}$.

e) Evaluate $\int_0^{\pi/2} \cos^9 x \, dx$.

f) Evaluate $\int_{\pi/4}^{\pi/2} \cot^7 x \, dx$.

g) Show that the spheres $x^2 + y^2 + z^2 + 6y + 14z + 28 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 24 = 0$ cut orthogonally.

h) Find the equation of a right circular cone with vertex at the origin, semi vertical angle 45° and axis along x-axis.

P.T.O.

PART – B

(3×5=15)

II. Answer **any three** questions :

a) Find the rank of the matrix, $A = \begin{pmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 8 \end{pmatrix}$ by row reduced Echelon form.

b) Solve the system of equations $x + 2y - z = 3$, $3x - y + 2z = 1$ and $2x - 2y + 3z = 2$ are consistent and solve them.

c) Reduce to normal form and hence find its rank :

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$$

d) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

e) Using Cayley-Hamilton theorem, find the inverse of the matrix.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

PART – C

(3×5=15)

III. Answer **any three** questions :

a) Find the n^{th} derivative of

i) $\log(x^2 - 9)$

ii) $e^x \sin x \cos 2x$.

b) Find the n^{th} derivative of $\frac{x-1}{(x-2)^2 (x+2)}$.

c) If $x = \sin t$, $y = \cos pt$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - p^2)y_n = 0$.

d) Verify that $\frac{\partial^2 z}{\partial x \, \partial y} = \frac{\partial^2 z}{\partial y \, \partial x}$ in the case when $z = \frac{xy}{y - x}$.

e) If $u = x^2 - 2y$, $v = x + y$, find $J = \frac{\partial(u, v)}{\partial(x, y)}$ and $J' = \frac{\partial(x, y)}{\partial(u, v)}$. Verify that $J.J' = 1$.

PART – D

(2×5=10)

IV. Answer **any two** questions :

a) Obtain the reduction formula for $\int \sin^n x \, dx$ where 'n' is a positive integer.

b) Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)4}$

c) Evaluate $\int_{\pi/4}^{\pi/2} \cot^7 x \, dx$.

PART – E

(2×5=10)

V. Answer **any two** questions :

a) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and has its centre lying on the plane $x + 4y + 3z + 7 = 0$.

b) Find the right circular cylinder generated by revolving the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$ about the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+5}{-1}$.

c) Find the equation of the right circular cone generated by revolving the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ about the line $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$.

PART – F

(2×5=10)

VI. Answer **any two** questions :

a) Find the instantaneous velocity, acceleration of the displacement of the particle $S = 3t^2 + 5$, at $t = 1$.

b) Find the value of three different objects x, y, z by reducing it into row reduced form, the system of their equations are given by $x + 2y - z = 3$, $3x - y + 2z = 1$ and $2x - 2y + 3z = 2$.

c) Find the equations of the spherical ball having the points (2, 1, –3) and (1, –2, 4) as the ends of a diameter. Find its centre and radius.