NP – 220

II

I Semester B.Sc. Examination, January/February 2025
(NEP) (Repeaters)
MATHEMATICS (Major)
Algebra – I and Calculus – I

Time : 2½ Hours   Max. Marks : 60

Instruction : *Answer **all** questions.*

PART – A

I. Answer **any four** questions. **(4×2=8)**

1) Define equivalent matrices.

2) Find the angle between the radius vector and the tangent to the curve $r = a\,(1 + \sin\theta)$ at $\theta = \frac{\pi}{6}$.

3) State Cauchy's mean value theorem.

4) Evaluate $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

5) State Leibnitz theorem.

6) Find the n^{th} derivative of $e^{2x}\sin 5x$.

PART – B

II. Answer **any four** questions. **(4×5=20)**

7) Find the inverse of the following matrices by using elementary transformation.

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

8) Solve the following system of equations $x + 2y + 3z = 0$, $2x + 3y + 4z = 0$, $7x + 13y + 19z = 0$.

9) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$ for the polar curve $r = f(\theta)$.

10) Find the angle of intersection of the curves $r = a\,(1 + \cos\theta)$ and $r = b\,(1 - \cos\theta)$.

11) Expand $\tan x$ by using Maclaurin's series upto the term containing x^4.

12) If $y = (x^2 - 1)^n$, show that $(x^2 - 1)\, y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

P.T.O.

PART – C

III. Answer **any four** questions. **(4×8=32)**

13) Verify Cayley Hamilton theorem for the following matrice and hence find its inverse.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

14) For what values of λ and μ the following system of equation has
   1) No solution
   2) Infinite number of solution
   3) Unique solution ?

   $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu.$

15) With usual notations prove that the radius of the curvature of the curve $y=f(x)$ is $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$.

16) Find all the asymptotes of the curve

   $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0.$

17) State and prove Rolle's theorem.

18) Trace the curve cardioid $r = a(1 + \cos\theta)$.