

**GN-231**

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III Semester B.A./B.Sc. Examination, December - 2019
(CBCS) (Semester Scheme) (F+R) (2015-16 and Onwards)

MATHEMATICS - III

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** questions.**PART - A**Answer **any five** questions.**5x2=10**

1. (a) Write the order of the elements of the group (\mathbb{Z}_4, t_4) .
- (b) Find all right cosets of the subgroup $\{0, 3\}$ in (\mathbb{Z}_6, t_6) .
- (c) Show that the sequence $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.
- (d) State Cauchy's root test for convergence.
- (e) Test the convergence of the series :

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \infty$$
- (f) Evaluate $\lim_{x \rightarrow \infty} x \cdot \sin\left(\frac{1}{x}\right)$.
- (g) State Cauchy's mean value theorem.
- (h) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$.

**PART - B**Answer **one** full question.**1x15=15**

2. (a) If a and b are any two arbitrary elements of a group G , then prove that $O(a) = O(b^{-1}ab)$.
- (b) If G is a group of fourth roots of unity and H is a subgroup of G , where $H = \{1, -1\}$ then write all cosets of H in G . Verify Lagrange's theorem.
- (c) State and prove Fermat's theorem in groups.

OR

3. (a) If a is a generator of a cyclic group G then prove that a^{-1} is also a generator.
- (b) In a group G , if $O(a) = n$, $\forall a \in G$, $d = (n, m)$, then prove that $O(a^m) = \frac{n}{d}$.
- (c) If G is a finite group and H is a subgroup of G then prove that order of H divides the order of G .

P.T.O.



PART - C

Answer **two** full questions.

2x15=30

4. (a) If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$, prove that $\lim_{n \rightarrow \infty} a_n \cdot b_n = ab$.
- (b) Discuss the nature of the sequence $\left\{ \frac{1}{n} \right\}$
- (c) Test the convergence of
- (i) $n[\log(n+1) - \log n]$
- (ii) $1 + \cos n\pi$

OR

5. (a) Prove that a monotonic decreasing sequence which is bounded below is convergent.
- (b) Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.
- (c) Examine the convergence of the sequence :
- (i) $\left\{ \frac{1 + (-1)^n n}{(n+1)} \right\}$
- (ii) $(2n+3) \cdot \sin\left(\frac{\pi}{n}\right)$

6. (a) Discuss the nature of the geometric series $\sum_{n=0}^{\infty} x^n$
- (b) Test the convergence of the series :
- $$1 + \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots$$
- (c) Sum the series to infinity
- $$\frac{1}{7} - \frac{1 \cdot 4}{7 \cdot 14} + \frac{1 \cdot 4 \cdot 7}{7 \cdot 14 \cdot 21} - \dots$$

OR

7. (a) State and prove Raabe's test for the convergence of series of positive terms.
- (b) Discuss the Leibnitz test on alternating series $\sum (-1)^{n-1} a_n$
- (c) Sum the series to infinity $\sum_{n=1}^{\infty} \frac{(n+1)(2n+1)}{(n+2)!}$

**PART - D**Answer **one** full question.**1x15=15**

8. (a) State and prove Lagrange's mean value theorem.

(b) Test the differentiability of $f(x) = \begin{cases} 1-3x, & x \leq 1 \\ x-3, & x > 1 \end{cases}$ at $x=1$.(c) Expand $\log_e(1 + \cos x)$ upto the term containing x^4 by using Maclaurin's series.**OR**

9. (a) Prove that a function which is continuous in closed interval takes every value between its bounds atleast once.

(b) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ by using Taylor's series expansion.Hence find the value of $\sin 91^\circ$ correct to 4 decimal places.(c) Evaluate : (i) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\log(\sin x)}{\left(\frac{\pi}{2} - x\right)^2}$ (ii) $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$

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