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III Semester B.Sc. Examination, March/April 2023
(NEP) (2022-23 and Onwards) (Freshers)
MATHEMATICS – III (Major)
Ordinary Differential Equations and Real Analysis – I

Time : 2½ Hours Max. Marks : 60

Instruction : *Answer **all** questions.*

PART – A

I. Answer **any four** questions. **(4×2=8)**

1) Test for exactness : $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^4)\,dy = 0$.

2) Find the general solution of $\sin px \cos y = \cos px \sin y + p$.

3) Solve : $\frac{d^4y}{dx^4} - 2\frac{d^2y}{dx^2} + y = 0$.

4) Show that $\left\{\frac{3n+5}{2n+1}\right\}$ is monotonic decreasing sequence.

5) Test the convergence of the series $\sum_{n=1}^{\infty}(-1)^n \cdot n$.

6) State Leibnitz test for alternating series.

PART – B

II. Answer **any four** questions. **(4×5=20)**

7) Solve : $xyp^2 - (x^2 + y^2)\,p + xy = 0$.

8) Solve : $y = 2px - yp^2$.

9) Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

10) If sequence $\{a_n\} \to a$ and sequence $\{b_n\} \to b$ as $n \to \infty$, prove that $\{a_n b_n\} \to ab$ as $n \to \infty$.

P.T.O.

11) Discuss the convergence of the sequence

$$a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!}.$$

12) Find the sum of the series :

$$1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \ldots\ldots$$

PART – C

III. Answer **any four** questions. **(4×8=32)**

13) Find the general and singular solution of $(px - y)(x - py) = 2p$ by using the substitution $x^2 = u$ and $y^2 = v$.

14) Solve : $\frac{d^2y}{dx^2} + 9y = \sec 3x$ by the method of variation of parameter.

15) Verify the condition for integrability and solve : $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})\,dz = 0$.

16) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

17) State and prove Cauchy's root test for the series of positive terms.

18) Sum to infinity the series :

$$1 - \frac{3}{5} + \frac{3.5}{5.10} - \frac{3.5.7}{5.10.15} + \ldots.\infty.$$