CS – 161

36

III Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2021 – 22 and Onwards) (Repeaters)
MATHEMATICS – III

Time : 3 Hours   Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

I. Answer **any five** questions. **(5×2=10)**

a) Find all the left cosets of $H = \{0,2,4\}$ of the group $(Z_6, +_6)$.

b) Define cyclic group.

c) Show that the sequence $\left\{\frac{3n+5}{2n+1}\right\}$ is monotonically decreasing sequence.

d) Discuss the convergence of the sequence $\left\{\frac{(n+1)^{n+1}}{n^n}\right\}$.

e) Examine the convergence of the series $\sum \frac{1}{3n-1}$.

f) State Raabe's test for convergence of series.

g) Find $L\left\{e^{5t} + 2e^{-3t}\right\}$.

h) Find $L^{-1}\left(\frac{5s}{s^2+9}\right)$.

PART – B

II. Answer **any two** questions. **(2×5=10)**

a) If a is any element of the group G, is of order n and e is the identity in G then prove that $a^n=e$, for any integer m, if and only if n divides m.

b) Prove that every subgroup of cyclic group is cyclic.

c) State and prove Lagranges theorem.

P.T.O.

PART – C

III. Answer **any two** questions. **(2×5=10)**

a) If $\{a_n\}$ and $\{b_n\}$ be two convergent sequence and $\lim\limits_{n\to\infty} a_n = l$ and $\lim\limits_{n\to\infty} b_n = m$, prove that $\lim\limits_{n\to\infty} a_n . b_n = l . m$.

b) Discuss the nature of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

c) Find the limit of the sequence 0.4,0.44,0.444,.....

PART – D

IV. Answer **any three** questions. **(3×5=15)**

a) Discuss the convergence of the series $\sum \frac{1.3.5....(2n-1)}{2.4.6.....2n} x^n$.

b) State and prove D'Alemberts ratio test for convergence of series of positive terms.

c) Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+....$

d) Sum to infinity of the series $\frac{1}{1.3}\left(\frac{1}{2}\right)+\frac{1}{2.3}\left(\frac{1}{2}\right)^2+\frac{1}{3.5}\left(\frac{1}{2}\right)^3+.....$

e) Sum to infinity of the series $\sum\limits_{n=1}^{\infty} \frac{3n^2-n+1}{n!}$.

PART – E

V. Answer **any three** questions. **(3×5=15)**

a) Evaluate $L\{\sin t . \sin 2t . \sin 3t\}$.

b) If $L\{f(t)\} = F(S)$, prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(S)$ ds and hence evaluate $L\left(\frac{\sin t}{t}\right)$.

c) Find the Laplace transform of the function

$$f(t) = \begin{cases} 1 & 0 < t < \frac{a}{2} \\ -1 & \frac{a}{2} < t < a \end{cases} \text{ and } f(t + a) = f(t).$$

d) Find $L^{-1}\left(\dfrac{4s + 5}{(s + 1)^2 (s + 2)}\right)$.

e) Using Convolution theorem, find the inverse Laplace transform of $\dfrac{1}{s(s^2 + 1)}$.

PART – F

VI. Answer **any two** questions. **(2×5=10)**

a) A farmer buys a used tractor for Rs. 12,000. He pays Rs. 6000 cash and agrees to pays the balance in annual installments of Rs.500 plus 12% interest on the unpaid amount. How much will be the tractor cost him ?

b) Solve $\dfrac{dy}{dt} - 5y = e^{5t}$, given $y(0) = 2$ using Laplace transform.

c) A person plucks 3 flowers on first day, doubles his plucking every day for about a year. If he does the work like this what is the number of flowers he was plucked on 365[th] day ?