CS – 164

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V Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2016 – 2017 and Onwards) (F+R)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

a) In a ring (R, +, .) show that $a\cdot(-b) = (-a)\cdot b = -(a\cdot b)\ \forall\ a, b \in R$.

b) Define field. Give an example.

c) Prove that every field is a principal ideal ring.

d) Find the divergence of the vector field.
$\vec{F} = x^3z\hat{i} + y^3x\hat{j} + z^3y\hat{k}$ at (1, 1, –1).

e) Find the maximum directional derivative of x sinz – y cosz at (0, 0, 0).

f) Prove that $E\nabla = \nabla E = \Delta$.

g) Evaluate $\Delta^{10}\ (1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)$.

h) State Simpson's $\frac{3}{8}^{th}$ rule for the integral $\int_a^b f(x)dx$.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) Prove that the intersection of any two subrings is a subring. Give an example to show that the union of 2 subrings of a ring need not be a subring.

b) Prove that $(Z_5, +_5, x_5)$ is a ring w.r.t. $+_5$ and x_5.

OR

P.T.O.

3. a) Prove that every field is an integral domain.

   b) Show that the set of all real numbers of the form $a+b\sqrt{2}$, where a and b are integers is a ring w.r.t. addition and multiplication.

4. a) If $f : R \to R'$ be a homomorphism and onto then prove that f is one-one iff $\ker f = \{ 0 \}$.

   b) Prove that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle/ a, b \in Z \right\}$ of all 2 × 2 matrices is a left ideal of the ring R over Z. Also show that S is not a right ideal.

OR

5. a) State and prove fundamental theorem of homomorphism of rings.

   b) Find all the principal ideals of the ring R = {0, 1, 2, 3, 4, 5, 6, 7} w.r.t. $\oplus_8$ and $\otimes_8$.

PART – C

Answer **any two full** questions : **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 4z^2$ at the point (1, 1, –8) in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

   b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 3$ at the point (2, –1, 2).

OR

7. a) Prove that $\nabla^2 r^n = n(n + 1) r^{n-2}$ where n is a non zero constant. Also deduce that r^n is harmonic if n = –1.

   b) If the vector $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal then find a.

8. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function. Then prove that $\text{div}(\phi\vec{F}) = \phi(\text{div}\,\vec{F}) + \nabla\phi\cdot\vec{F}$.

   b) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational, find ϕ such that $\vec{F} = \nabla\phi$.

OR

9. a) Prove that

   1) curl $\vec{F}$ is solenoidal

   2) grad ϕ is irrotational.

   b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) By the separation of symbols, prove that

$$u_0 + \frac{u_1}{1!} + \frac{u_2 x^2}{2!} + \ldots \infty = e^x \left[u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \ldots \infty \right].$$

   b) Obtain the function whose first difference is $6x^2 + 10x + 11$.

OR

11. a) From the following data find θ at x = 84 using difference table.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

   b) Express $3x^3 - 4x^2 + 3x - 11$ in factorial notation. Also express its successive difference in factorial notation.

12. a) Prepare divided difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920

   b) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simpson's $\frac{3}{8}^{th}$ rule.

OR

13. a) By using Lagrange's Interpolation formula, find f(10) from the following data.

x	5	6	9	11
f(x)	12	13	14	16

   b) Evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub intervals by using Simpson's $\frac{1}{3}^{rd}$ rule.