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V Semester B.A./B.Sc. Examination, March 2023
(CBCS) (2016 – 17 and Onwards) (F+R)
MATHEMATICS (Paper – VI)

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

1. Answer **any 5** questions. **(5×2=10)**

   a) Define geodesic on a surface.

   b) Find the function y which makes the integral $I = \int_{x_1}^{x_2} (1 + xy' + xy'^2)\, dx$ an extremum.

   c) Find the Euler's equation when f does not contain 'y' explicitly.

   d) Evaluate $\int_c (3x + y)dx + (2y - x)\, dy$ along y = x from (0, 0) to (10, 10).

   e) Evaluate $\int_0^a \int_0^b (x^2 + y^2) dxdy$.

   f) Evaluate $\int_0^1 \int_0^x \int_0^z dydzdx$.

   g) State stoke's theorem.

   h) Find the area of the circle $x^2 + y^2 = a^2$ by double integration.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Derive the Euler's equations in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

   b) Find the extremal of the functional $\int_{x_1}^{x_2} y'^2 - y^2 + 2y \sec x)dx$.

OR

3. a) Find the Geodesics on a surface given that the arc length on the surface $S = \int_{x_1}^{x_2} \sqrt{x(1 + y'^2)}\; dx$.

   b) Find the path in which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity.

4. a) Find the shape of a chain which hangs under gravity between two fixed points.

   b) Find the extremal of the functional $\int_0^1 (y'^2 + x^2)\,dx$ subject to the constraint $\int_0^1 y\,dx = 2$ and having the conditions y(0) = 0, y(1) = 1.

OR

5. a) Find the extremal of the integral $I = \int_0^1 y'^2\,dx$ subject to the constraint $\int_0^1 y\,dx = 1$ and having y(0) = 0, y(1) = 1.

   b) Find the equation of the curve which joins the points (0, 1) and (2, 3) and along which the integral $\int_0^2 \frac{\sqrt{1+y'^2}}{y}\,dx$ is a minimum.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Compute $\int_c x\,dx - y\,dx$ around the square (0, 0), (1, 0), (1, 1), (0, 1).

   b) Evaluate $\iint_c xy\,(x+y)\,dxdy$ over the region R bounded between the parabola $y = x^2$ and the line y = x.

OR

7. a) Change the order of integration in $\int_0^a \int_0^{2\sqrt{ax}} x^2\,dxdy$ and hence evaluate.

   b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dzdydx$.

   b) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} x^2\,dydx$ by changing to polar coordinates.

OR

9. a) Find the value of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrations.

   b) Evaluate $\iiint_R xyz\,dx\,dydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into cylindrical polar co-ordinates.

PART – D

Answer **any two full** questions. **(2×10=20)**

10. a) State and prove Green's theorem.

b) Using divergence theorem, evaluate $\iint \vec{F}.\hat{n}ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + x^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$.

OR

11. a) Using divergence theorem evaluate $\iint_s (x\hat{i} + y\hat{j} + z^2\hat{k}).\hat{n}ds$ where s is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$

b) Using Green's theorem evaluate $\int_c e^{-x}\sin y dx + e^{-x}\cos y dy$ where C is the rectangle with the vertices $(0, 0)$, $(0, \pi/2)$ $(\pi, \pi/2)$, $(\pi, 0)$.

12. a) Verify stoke's theorem for the function $\vec{F} = y^2\hat{i} + xy\hat{j} - xz\hat{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

b) State and prove Gauss-divergence theorem.

OR

13. a) Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$.

b) Evaluate by Stoke's theorem $\int_c \sin z dx - \cos x dy + \sin y dz$, where 'C' is the boundary of the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $z = 3$