NP – 226

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V Semester B.Sc. Examination, January/February 2025
(NEP) (Freshers/Repeaters)
MATHEMATICS (Major)
Paper – 5.1 : Real Analysis – II and Complex Analysis

Time : 2½ Hours   Max. Marks : 60

Instruction : *Answer **all** questions.*

PART – A

I. Answer **any ten** questions : **(10×2=20)**

1) Define partition of closed interval.

2) Show that $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{2n}\right]=\frac{\pi}{4}$.

3) Define common refinement.

4) Evaluate $\beta\left(\frac{9}{2},\frac{7}{2}\right)$.

5) Define convergence at both the end points.

6) Prove that $\Gamma(n+1) = n\Gamma(n)$.

7) Define Analytic function.

8) Find the locus of the point z satisfying $|z+i| \le 2$.

9) Show that $f(z) = z^2 + 2z$ is continuous at $1+i$.

10) State Cauchy's integral formula.

11) Evaluate $\int_C \frac{e^z}{z}\,dz$, where C is the unit circle with centre at the origin.

12) Find the fixed points of the transformation $W=\frac{3z-4}{z}$.

P.T.O.

PART – B

II. Answer **any two** questions : **(2×5=10)**

13) Show that f(x) = sinx is Riemann integrable in [a, b].

14) Show that every continuous function is Riemann integrable.

15) If f and g are Riemann integrable over [a, b], show that f+g is also Riemann integrable over [a, b].

16) Show that by using Riemann Integrable, $\int_1^2 f(x)dx = \frac{27}{2}$ where f(x) = 5x + 6.

PART – C

III. Answer **any two** questions : **(2×5=10)**

17) Test the convergence of $\int_0^1 \frac{dx}{2x - x^2}$.

18) Prove that $\beta(m, n) \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, where m, n > 0.

19) Prove that $\int_0^{\pi/2} \tan\theta d\theta = \frac{\pi}{\sqrt{2}}$.

20) Show that improper integral of $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if n < 1.

PART – D

IV. Answer **any two** questions : **(2×5=10)**

21) Prove that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

22) If f(z) = u + iv is analytic, then prove that $\frac{\partial}{\partial x}|f(z)|^2 + \frac{\partial}{\partial y}|f(z)|^2 = |f'(z)|^2$.

23) Show that f(z) = logz is analytic and hence find $f'(z)$.

24) Find the analytic, function f(z) = u + iv whose imaginary part is xsinx sinhy – ycosx coshy.

PART – E

V. Answer **any two** questions : **(2×5=10)**

25) Evaluate $\int_c \frac{dz}{z^2 - 4}$ over (i) c : |z| = 1 (ii) c : |z + 2| = 1.

26) State and prove Cauchy's integral theorem.

27) Discuss the transformation $w = \frac{1}{z}$.

28) Find the bilinear transformation which maps the points z = 1, i, – 1 into w = 0, 1, ∞.