NP – 227

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V Semester B.Sc. Examination, Jan./Feb. 2025
(NEP) (Freshers/Repeaters)
MATHEMATICS (Major)
Paper – 5.2 : Vector Calculus and Analytical Geometry

Time : 2½ Hours | Max. Marks : 60

***Instruction** : Answer **all** questions.*

PART – A

I. Answer **any ten** questions : **(10×2=20)**

1) If $\vec{r} = 3t^2 + 2t + 1$, find $\left|\frac{d^2\vec{r}}{dt^2}\right|$ at t = 1.

2) If $\vec{F} = (ax + 3y + 4z)\,\hat{i} + (x - 2y + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$ is solenoidal. Find 'a'.

3) Find the maximum directional derivative of $\phi = 4x^3y^2z$ at the point (1, – 2, 4).

4) State Gauss divergence theorem.

5) Write vector form of Green's theorem.

6) Evaluate by Stoke's theorem $\oint_C [yzdx + zxdy + xydz]$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

7) Find the angle between the planes $2x + 4y - 6z = 1$ and $3x + 6y + 5z + 4 = 0$.

8) Find the equation of the line passing through the point (1, 1, 1) and (2, 3, 4).

9) Find the equation of the plane containing the point (2, 1, 1) and the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+1}{-1}$.

10) Find the equation of the sphere whose ends of the diameters are (1, 1, 2) and (2, 1, 2).

11) Define the pole and polar plane.

12) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 6x - 8y - 2z - 1 = 0$.

P.T.O.

PART – B

II. Answer **any two** questions : **(2×5=10)**

13) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, – 1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

14) Show that $\vec{F} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\,\hat{k}$ is irrotational. Also find ϕ such that $\nabla\phi = \vec{F}$.

15) If $\vec{F} = x^3 + y^3 + z^3 - 3xyz$, find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at the point (1, 1, 1).

16) Prove that

i) $\nabla \times (\nabla\phi) = 0$.

ii) $\nabla.(\nabla \times \vec{F}) = 0$.

PART – C

III. Answer **any two** questions : **(2×5=10)**

17) State and prove Green's theorem.

18) By using Green's theorem, evaluate $\oint_C (3x - y)\,dx + (2x + y)\,dy$, where C is the circle $x^2 + y^2 = a^2$.

19) Using Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = (x^2 - y^2)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$ over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

20) Evaluate by Stoke's theorem $\oint_C [(y - z + 2)\,dx + (yz + 4)\,dy - xz\,dz]$ and C is the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

PART – D

IV. Answer **any two** questions : **(2×5=10)**

21) Find the equation of the plane which bisects the angle between the plane $3x - 4y + 5z - 3 = 0$ and $5x + 3y - 4z - 9 = 0$.

22) Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$.

23) Find the angle between the line $\frac{x-3}{2} = \frac{y+1}{-1} = \frac{z+3}{3}$ and the plane $2x + 3y - z - 4 = 0$.

24) Find the equation of the sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and having its centre on the plane $x + 2y - 3z - 4 = 0$.

PART – E

V. Answer **any two** questions : **(2×5=10)**

25) Derive the equation of the right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2\alpha$.

26) Find the equation of the right circular cone whose vertex is (2, 3, 5), axis makes equal angles with the coordinate axes and the semivertical angle is $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$.

27) Explain the equation of ellipsoid with properties.

28) Find the equation of a cylinder whose generators touch the sphere $x^2 + y^2 + z^2 = 9$ and having its generator parallel to the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.