



NP – 227

31.

V Semester B.Sc. Examination, Jan./Feb. 2025

(NEP) (Freshers/Repeaters)

MATHEMATICS (Major)

Paper – 5.2 : Vector Calculus and Analytical Geometry

Time : 2½ Hours

Max. Marks : 60

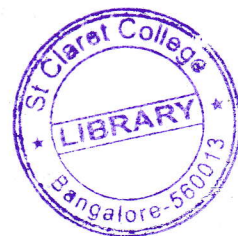
Instruction : Answer **all** questions.

PART – A

I. Answer **any ten** questions :

(10×2=20)

- 1) If $\vec{r} = 3t^2 + 2t + 1$, find $\left| \frac{d^2\vec{r}}{dt^2} \right|$ at $t = 1$.
- 2) If $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal. Find 'a'.
- 3) Find the maximum directional derivative of $\phi = 4x^3y^2z$ at the point $(1, -2, 4)$.
- 4) State Gauss divergence theorem.
- 5) Write vector form of Green's theorem.
- 6) Evaluate by Stoke's theorem $\oint_C [yzdx + zxdy + xydz]$ where C is the curve $x^2 + y^2 = 1, z = y^2$.
- 7) Find the angle between the planes $2x + 4y - 6z = 1$ and $3x + 6y + 5z + 4 = 0$.
- 8) Find the equation of the line passing through the point $(1, 1, 1)$ and $(2, 3, 4)$.
- 9) Find the equation of the plane containing the point $(2, 1, 1)$ and the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+1}{-1}$.
- 10) Find the equation of the sphere whose ends of the diameters are $(1, 1, 2)$ and $(2, 1, 2)$.
- 11) Define the pole and polar plane.
- 12) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 6x - 8y - 2z - 1 = 0$.



P.T.O.



PART – B

II. Answer **any two** questions :

(2×5=10)

- 13) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.
- 14) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Also find ϕ such that $\nabla\phi = \vec{F}$.
- 15) If $\vec{F} = x^3 + y^3 + z^3 - 3xyz$, find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at the point $(1, 1, 1)$.
- 16) Prove that
- $\nabla \times (\nabla\phi) = 0$.
 - $\nabla \cdot (\nabla \times \vec{F}) = 0$.

PART – C

III. Answer **any two** questions :

(2×5=10)

- 17) State and prove Green's theorem.
- 18) By using Green's theorem, evaluate $\oint_C (3x - y) dx + (2x + y) dy$, where C is the circle $x^2 + y^2 = a^2$.
- 19) Using Gauss divergence theorem to evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = (x^2 - y^2)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelopiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.
- 20) Evaluate by Stoke's theorem $\oint_C [(y - z + 2) dx + (yz + 4) dy - xzdz]$ and C is the boundary of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.



PART - D

IV. Answer **any two** questions :

(2×5=10)

- 21) Find the equation of the plane which bisects the angle between the plane $3x - 4y + 5z - 3 = 0$ and $5x + 3y - 4z - 9 = 0$.
- 22) Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$.
- 23) Find the angle between the line $\frac{x-3}{2} = \frac{y+1}{-1} = \frac{z+3}{3}$ and the plane $2x + 3y - z - 4 = 0$.
- 24) Find the equation of the sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and having its centre on the plane $x + 2y - 3z - 4 = 0$.

PART - E

V. Answer **any two** questions :

(2×5=10)

- 25) Derive the equation of the right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.
- 26) Find the equation of the right circular cone whose vertex is (2, 3, 5), axis makes equal angles with the coordinate axes and the semivertical angle is $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$.
- 27) Explain the equation of ellipsoid with properties.
- 28) Find the equation of a cylinder whose generators touch the sphere $x^2 + y^2 + z^2 = 9$ and having its generator parallel to the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.
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