

II Semester B.Sc. Examination, Sept./Oct. 2022 (NEP – 2021-2022 and Onwards) MATHEMATICS

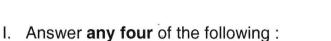
DSC - 2.1 : Algebra - II and Calculus - II

Time: 21/2 Hours

Max. Marks: 60

 $(4 \times 2 = 8)$

PART - A



1) State Fermat's theorem.

2) If
$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 3 & 4 & 1 \end{pmatrix}$$
, $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 6 & 5 \end{pmatrix}$ find $(f \circ g)^{-1}$.

- 3) Show that $f: (G, +) \rightarrow (G', +)$ defined by $f(x) = x^2$ is not a homomorphism.
- 4) If $z = e^{2x} \sin 3y$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
- 5) Show that $f(x, y) = x^3 + y^3 3xy + 1$ is minimum at (1, 1).
- 6) Evaluate $\iint_{0}^{1} \iint_{0}^{2} xyz^{2} dx dy dz.$

PART - B

II. Answer any four of the following:

 $(4 \times 5 = 20)$

- 7) Find all the left cosets of the subgroup $H = \{0, 3, 6, 9\}$ of the group $(Z_{12}, +_{12})$.
- 8) State and prove Lagrange's theorem.
- 9) If $f: G \to G'$ is a homomorphism from the group G into G' and H is a subgroup of G, then prove that f(H) is again a subgroup of G'.
- 10) If $u = sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = tanu$.
- 11) Evaluate $\int_C (xy dx + x^2 z dy + xyz dz)$ where C is given by $x = e^t$, $y = e^{-t}$, $z = t^2$ and $0 \le t \le 1$.
- 12) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1$ by changing into polar co-ordinates.

P.T.O.

PART - C

III. Answer any four of the following:

 $(4 \times 8 = 32)$

- 13) Define the order of an element of a group. In a group G, prove that $O(a) = O(a^{-1}), \forall a \in G$.
- 14) Define normal subgroup of a group. Prove that a subgroup H of a group G is normal iff $gHg^{-1} = H$, $\forall g \in G$.
- Define factor group. State and prove the fundamental theorem of homomorphism.
- 16) Obtain Taylor's expansion of $f(x, y) = e^x \log(1+y)$ about the point (0, 0) upto the third degree term.
- 17) If u = x + y, v = x y find $J = \frac{\partial(u, v)}{\partial(x, y)}$, $J' = \frac{\partial(x, y)}{\partial(u, v)}$ and verify that JJ' = 1.
- 18) Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} \frac{dz \, dy \, dx}{\sqrt{1-x^2-y^2-z^2}}$ by changing into spherical polar coordinates.