NP – 103

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II Semester B.Sc. Examination, Sept./Oct. 2022
(NEP – 2021-2022 and Onwards)
MATHEMATICS
DSC – 2.1 : Algebra – II and Calculus – II

Time : 2½ Hours Max. Marks : 60

PART – A

I. Answer **any four** of the following : **(4×2=8)**

1) State Fermat's theorem.

2) If $f=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 5 & 3 & 4 & 1\end{pmatrix}, g=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 1 & 2 & 4 & 6 & 5\end{pmatrix}$ find $(f\circ g)^{-1}$.

3) Show that $f : (G, +) \to (G', +)$ defined by $f(x) = x^2$ is not a homomorphism.

4) If $z = e^{2x} \sin 3y$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

5) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at (1, 1).

6) Evaluate $\int_0^1\int_0^2\int_1^2 xyz^2 \, dx\, dy\, dz$.

PART – B

II. Answer **any four** of the following : **(4×5=20)**

7) Find all the left cosets of the subgroup H = {0, 3, 6, 9} of the group $(Z_{12}, +_{12})$.

8) State and prove Lagrange's theorem.

9) If $f : G \to G'$ is a homomorphism from the group G into G′ and H is a subgroup of G, then prove that f(H) is again a subgroup of G′.

10) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

11) Evaluate $\int_C (xy\,dx + x^2 z\,dy + xyz\,dz)$ where C is given by $x = e^t$, $y = e^{-t}$, $z = t^2$ and $0 \le t \le 1$.

12) Evaluate $\int\int xy \, dx\, dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1$ by changing into polar co-ordinates.

P.T.O.

PART – C

III. Answer **any four** of the following : **(4×8=32)**

13) Define the order of an element of a group. In a group G, prove that $O(a) = O(a^{-1}), \forall a \in G$.

14) Define normal subgroup of a group. Prove that a subgroup H of a group G is normal iff $gHg^{-1} = H, \forall g \in G$.

15) Define factor group. State and prove the fundamental theorem of homomorphism.

16) Obtain Taylor's expansion of $f(x, y) = e^x \log(1+y)$ about the point (0, 0) upto the third degree term.

17) If $u = x + y$, $v = x - y$ find $J = \frac{\partial(u, v)}{\partial(x, y)}$, $J' = \frac{\partial(x, y)}{\partial(u, v)}$ and verify that $JJ' = 1$.

18) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$ by changing into spherical polar coordinates.