



IV Semester B.A./B.Sc. Examination, Sept./Oct. 2022  
(Semester Scheme) (CBCS) (2021-22 and Onwards) (F+R)  
MATHEMATICS – IV

Time : 3 Hours

Max. Marks : 70

*Instruction : Answer all Parts.*

## PART – A

1. Answer any five questions.

(5×2=10)

- Define Isomorphism of groups.
- Prove that every subgroup of an abelian group is normal.
- Define Fourier series.
- Verify Rolle's theorem for  $f(x) = x^2 - 6x + 8$  in  $[2, 4]$ .
- Evaluate :  $\lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{x^2} \right)$ .
- Show that  $f(x, y) = x^3 + y^3 - 3xy + 1$  is minimum at the point  $(1, 1)$ .
- Solve  $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 0$ .
- Find the particular integral of  $(D^3 + D^2 + D + 1)y = e^{3x+4}$ .

## PART – B

2. Answer any two questions.

(2×5=10)

- Prove that a subgroup  $H$  of a group  $G$  is normal if and only if  $ghg^{-1} \in H$ ,  $\forall g \in G$ .
- Prove that a subgroup  $H$  of a group  $G$  is normal if and only if every right coset of  $H$  in  $G$  is a left coset of  $H$  in  $G$ .
- State and prove Fundamental theorem of Homomorphism.



## PART – C

3. Answer **any two** questions.

(2×5=10)

a) Obtain the Fourier series for  $f(x) = x^2$  in  $-\pi < x < \pi$  and hence deduce

$$\text{that } \frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

b) Find the Fourier series for the function

$$f(x) = \begin{cases} x + \frac{\pi}{2}, & -\pi < x < 0 \\ \frac{\pi}{2} - x, & 0 \leq x < \pi \end{cases}$$

c) Find the half range sine series for  $f(x) = 2x - 1$  over  $(0, 2)$ .

## PART – D

4. Answer **any three** questions.

(3×5=15)

a) Examine the differentiability of

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

b) State and prove Cauchy's mean value theorem.

c) Evaluate :

i)  $\lim_{x \rightarrow 0} \log_{\sin x} \sin 2x$

ii)  $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$ .

d) Obtain Maclaurin's expansion of the function  $e^{\sin x}$ .

e) Find the extreme values of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$

## PART – E

5. Answer **any three** questions.

(3×5=15)

a) Solve  $y'' + 3y' + 2y = \cos^2 x$ .

b) Solve  $(D^2 - 3D + 2)y = e^x \sin x$ .



- c) Solve  $x^2 D^2 - 2x(x+1)D + 2(x+1)y = x^3$  given that  $x$  is a part of complementary function.
- d) Solve  $\cos xy'' + \sin xy' - 2 \cos^2 xy = 2 \cos^5 x$ .
- e) Solve  $\frac{d^2y}{dx^2} + y = \sec x$  by the method of variation of parameters.

PART – F

6. Answer **any two** questions.

**(2×5=10)**

- a) Sketch the graph of triangular wave and explain it.
  - b) Find the Fourier series representing  $f(x) = x$ ,  $0 < x < 2\pi$  and sketch its graphs from  $x = -4\pi$  to  $x = 4\pi$ .
  - c) Sketch the graph of output voltage in Half-wave rectifier and explain it.
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