UG – 170

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IV Semester B.A./B.Sc. Examination, September/October 2022
(Semester Scheme)
(CBCS) (F+R) (2015 – 16 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** Parts.*

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PART – A

1. Answer **any five** questions : **(5×2=10)**

a) Define homomorphism and isomorphism of a group.

b) Define centre of a group.

c) Write the formula for b_n of Fourier sine series expansion.

d) Find the critical points of the function
$f(x, y) = 2x^2 - xy + y^2 + 7x$.

e) Find $L^{-1}\left\{\dfrac{5s}{s^2+9}\right\}$.

f) Find $L\{e^{3t}\sin 5t\}$.

g) Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 8y = 0$.

h) Find the complementary function of $(D^2 - 4)y = 0$.

PART – B

Answer **any one full** question : **(1×15=15)**

2. a) Show that a subgroup H of a group G is normal subgroup iff $gHg^{-1} = H$, $\forall\, g \in G$.

b) Let $f : G \to G'$ be a homomorphism from the group G into G′ with Kernel K, then show that f is one-one if and only if $K = \{e\}$ where e is the identity element of G.

c) Prove that the centre of a group G is normal subgroup of G.

OR

P.T.O.

3. a) State and prove fundamental theorem of homomorphism.
   b) Prove that every group of a cyclic group is cyclic.
   c) If $f : G \to G$ be a homomorphism of a group G into itself and H is a cyclic subgroup of G then prove that f(H) is also cyclic.

PART – C

Answer **any two full** questions : **(2×15=30)**

4. a) Obtain the Fourier series for the function $f(x) = x^2$ over the interval $(-\pi, \pi)$.
   b) Obtain the half range cosine series for $f(x) = x$ in the interval $0 < x < \pi$.
   c) Expand $e^{ax}\cos by$ in Taylor's series upto second degree terms about the origin.

OR

5. a) Find the extreme value of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
   b) A rectangular box, open at the top, is to have a volume of 32 cubic units, find the dimensions so that the total surface is a minimum.
   c) Obtain the half range Fourier sine series of $f(x) = (x - 1)^2$ in the interval (0, 1).

6. a) Find L{sint sin2t sin3t}.
   b) Find the Laplace transform of the function $(3t^2 + 4t + 5)(t - 3)$.
   c) Find $L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\right\}$.

OR

7. a) Find $L\left\{\frac{\cos 2t - \sin 3t}{t}\right\}$.
   b) Verify convolution theorem for the function $f(t) = \sin t$, $g(t) = e^{-t}$.
   c) Find $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$.

PART – D

Answer **any one full** question : **(1×15=15)**

8. a) Solve $(D^2 - 2D + 1)y = \sinh x$.

   b) Solve $(D^2 + 4)y = \sin^2 x$.

   c) Solve $(D^2 + D - 6)y = x$.

OR

9. a) Solve $(D^2 - 2D + 4)y = e^x \cos x$.

   b) Solve $\frac{dy}{dt} = 3x - y$; $\frac{dy}{dt} = x + y$.

   c) Solve $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin(x^2)$ using the transformation $z = x^2$.