



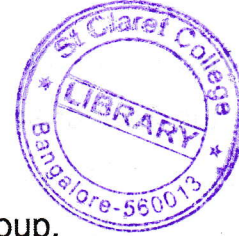
IV Semester B.A./B.Sc. Examination, September/October 2022  
(Semester Scheme)  
(CBCS) (F+R) (2015 – 16 and Onwards)  
MATHEMATICS (Paper – IV)

Time : 3 Hours

Max. Marks : 70

**Instruction : Answer all Parts.**

## PART – A

1. Answer **any five** questions :

(5×2=10)

- Define homomorphism and isomorphism of a group.
- Define centre of a group.
- Write the formula for  $b_n$  of Fourier sine series expansion.
- Find the critical points of the function  
 $f(x, y) = 2x^2 - xy + y^2 + 7x$ .
- Find  $L^{-1} \left\{ \frac{5s}{s^2 + 9} \right\}$ .
- Find  $L\{e^{3t} \sin 5t\}$ .
- Solve  $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$ .
- Find the complementary function of  $(D^2 - 4)y = 0$ .

## PART – B

Answer **any one full** question :

(1×15=15)

- Show that a subgroup  $H$  of a group  $G$  is normal subgroup iff  $gHg^{-1} = H$ ,  $\forall g \in G$ .
  - Let  $f : G \rightarrow G'$  be a homomorphism from the group  $G$  into  $G'$  with Kernel  $K$ , then show that  $f$  is one-one if and only if  $K = \{e\}$  where  $e$  is the identity element of  $G$ .
  - Prove that the centre of a group  $G$  is normal subgroup of  $G$ .

OR

P.T.O.



3. a) State and prove fundamental theorem of homomorphism.  
 b) Prove that every group of a cyclic group is cyclic.  
 c) If  $f : G \rightarrow G$  be a homomorphism of a group  $G$  into itself and  $H$  is a cyclic subgroup of  $G$  then prove that  $f(H)$  is also cyclic.

## PART – C

Answer any two full questions :

(2×15=30)

4. a) Obtain the Fourier series for the function  $f(x) = x^2$  over the interval  $(-\pi, \pi)$ .  
 b) Obtain the half range cosine series for  $f(x) = x$  in the interval  $0 < x < \pi$ .  
 c) Expand  $e^{ax}\cos bx$  in Taylor's series upto second degree terms about the origin.

OR

5. a) Find the extreme value of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .  
 b) A rectangular box, open at the top, is to have a volume of 32 cubic units, find the dimensions so that the total surface is a minimum.  
 c) Obtain the half range Fourier sine series of  $f(x) = (x - 1)^2$  in the interval  $(0, 1)$ .
6. a) Find  $L\{\sin t \sin 2t \sin 3t\}$ .  
 b) Find the Laplace transform of the function  $(3t^2 + 4t + 5)(t - 3)$ .  
 c) Find  $L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\right\}$ .

OR

7. a) Find  $L\left\{\frac{\cos 2t - \sin 3t}{t}\right\}$ .  
 b) Verify convolution theorem for the function  $f(t) = \sin t$ ,  $g(t) = e^{-t}$ .  
 c) Find  $L^{-1}\left[\log\left(\frac{s^2 + 1}{s(s+1)}\right)\right]$ .



PART – D

Answer **any one full** question :

(1×15=15)

8. a) Solve  $(D^2 - 2D + 1)y = \sinh x$ .

b) Solve  $(D^2 + 4)y = \sin^2 x$ .

c) Solve  $(D^2 + D - 6)y = x$ .

OR

9. a) Solve  $(D^2 - 2D + 4)y = e^x \cos x$ .

b) Solve  $\frac{dy}{dt} = 3x - y$ ;  $\frac{dy}{dt} = x + y$ .

c) Solve  $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$  using the transformation  $z = x^2$ .

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