NP – 817

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VI Semester B.Sc. Examination, June/July 2025
(NEP Scheme) (F+R)
MATHEMATICS (Major)
DSC 6.1 : Rings, Fields and Linear Algebra

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer **all** Parts.*

PART – A

I. Answer **any ten** questions. **(10×2=20)**

1) Define a ring and give an example.

2) Show that the set $M = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}; a, b \in R \right\}$ is a subring of the ring R of all 2 × 2 matrices over the field of real numbers.

3) Define maximal ideal of a ring R and give an example.

4) Prove that in any vector space V over a field F, $a\alpha = 0 \Rightarrow a = 0$ or $\alpha = 0$ $\forall\, a \in F$ and $\forall\, \alpha \in V$.

5) Show that $W = \{(x, y, z)/2x + 3y + z = 0\}$ is a subspace of $V_3(R)$.

6) Express the vector (3, 5, 2) as a linear combination of (1, 1, 0), (2, 3, 0), (0, 0, 1).

7) If $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x + y, y)$, then show that T is a linear transformation.

8) For a matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$, find the linear transformation corresponding to standard bases.

9) Find the range space of the linear transformation $T : R^2 \to R^2$ defined by $T(x, y) = (x + y, x)$.

10) Define isomorphism of a transformation.

P.T.O.

11) Find the eigen values of a linear transformation $T : V_2(R) \to V_2(R)$ defined by T(1, 0) = (1, 2), T(0, 1) = (4, 3).

12) Define Diagonalizable matrix.

PART – B

II. Answer **any two** questions. **(2×5=10)**

13) Prove that the set R = {0, 1, 2, 3, 4} is a commutative ring with unity w.r.t $+_5$ and $\times_5$ as the two ring compositions.

14) Prove that the intersection of any two subrings is a subring. Is the union is also a subring. Justify your answer.

15) Prove that the ring $(Z_n, +_n, \times_n)$ is an integral domain if and only if n is a prime number.

16) State and prove fundamental theorem of homomorphism on rings.

PART – C

III. Answer **any two** questions. **(2×5=10)**

17) Show that $V = \{a + b\sqrt{2} \,/\, a, b \in Q\}$ is a vector space over Q, where Q is the set of rationals.

18) Show that the vectors (1, –2, 5) is a linear combination of the vectors (1, 1, 1), (1, 2, 3) and (2, –1, 1).

19) Prove that a set of non-zero vectors $\alpha_1, \alpha_2 \ldots \alpha_n$ of a vector space V(F) is linearly dependent if and only if some one of those vectors say α_k $(2 \le k \le n)$ is expressed as a linear combination of its preceding ones.

20) Show that the vectors (2, 1, 4), (1, –1, 2), (3, 1, –2) form a basis of R^3.

PART – D

IV. Answer **any two** questions. **(2×5=10)**

21) Define linear transformation. If $T : U \to V$ be a linear transformation, then prove that

i) T(0) = 0′ where 0 and 0′ are zero vectors of U and V respectively.

ii) $T(-\alpha) = -T(\alpha)\ \forall\ \alpha \in U$.

22) Find a linear transformation $T : R^2 \to R^3$ such that $T(-1, 1) = (-1, 0, 2)$, $T(2, 1) = (1, 2, 1)$.

23) Find a matrix of linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3)$ w.r.t the bases $B_1 = \{(1, 1, 0), (1, 0, 1), (1, 1, -1)\}$ and $B_2 = \{(2, -3), (1, 4)\}$.

24) State and prove rank-nullity theorem.

PART – E

V. Answer **any two** questions. **(2×5=10)**

25) Show that the correspondence $(x, y, z) \to (-x, y, z)$ is an automorphism of the vector space $V_3(R)$ and find its order.

26) State and prove fundamental theorem of homomorphism of a transformation.

27) Find the eigen values and eigen vectors of a linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (3x + y, 6x + 2y)$.

28) Show that the matrix $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$ is diagonalizable.